

The University of Calgary  
 Department of Mathematics and Statistics  
 MATH 249/01 Quiz # 5W Fall 2010

Name: \_\_\_\_\_ I.D.#: \_\_\_\_\_

1. Solve for  $x$ :  $3^{2x} = \frac{7}{9^{x+2}}$ . [3]

2. Find the domain and derivative of  $y = \left(\frac{1}{x}\right)^{x^2} + \ln(1-x)$ . [4]

3. If 80 kg of radon-x decayed to 50 kg in 3 days  
 how much will be remaining after 5 days? [3]

**Solution**

**For 1)**

apply ln to both sides:  $\ln(3^{2x}) = \ln\left(\frac{7}{9^{x+2}}\right)$

use properties of ln  $2x \ln 3 = \ln 7 - (x+2) \ln 9$

$2x \ln 3 = \ln 7 - x \ln 9 - 2 \ln 9$   $x(2 \ln 3 + \ln 9) = \ln 7 - 2 \ln 9$

$x \ln(9 \cdot 9) = \ln \frac{7}{9^2}$   $x = \frac{\ln(7/81)}{\ln 81}$

**For 2)**

$y = \left(\frac{1}{x}\right)^{x^2} + \ln(1-x) = e^{x^2 \ln \frac{1}{x}} + \ln(1-x) = e^{-x^2 \ln x} + \ln(1-x)$

for  $x > 0$  and  $1-x > 0$   $D = (0, 1)$  --- domain

$y' = [e^{-x^2 \ln x}]' + [\ln(1-x)]' = e^{-x^2 \ln x} (-x^2 \ln x)' + \frac{1}{1-x} \cdot (1-x)'$   
 $= e^{-x^2 \ln x} (-x^2 \ln x)' - \frac{1}{1-x} = e^{-x^2 \ln x} (-2x \ln x - x^2 \cdot \frac{1}{x}) - \frac{1}{1-x} =$   
 $= -xe^{-x^2 \ln x} (2 \ln x + 1) - \frac{1}{1-x}$

**For 3)**

the correct formula for the amount left  $A(t) = A_0 e^{kt}$  where  $k < 0$

$t$  in days,  $A_0 = 80 \text{ kg}$  for  $t = 3$   $50 = 80e^{3k}$   $\left(\frac{5}{8}\right)^{\frac{1}{3}} = e^k$

solve for  $k$   $\ln \frac{50}{80} = 3k$   $k = \frac{\ln 5/8}{3} = -0.1566$

then for  $t = 5$   $A = 80e^{5k} = 36.55$

also  $A = 80 \left(e^k\right)^5 = 80 \left(\frac{5}{8}\right)^{\frac{5}{3}}$