THE UNIVERSITY OF CALGARY
MATHEMATICS 249
FINAL EXAMINATION, FALL 2006
TIME: 2 HOURS

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Total (max. 75)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [75]. THIS EXAM HAS 8 PAGES INCLUDING THIS ONE.

[5] 1. Find $\lim_{x \to 2} \left(\frac{5}{x-2} - \frac{7x-4}{x^2-2x} \right)$.

[5] 2. Find $\lim_{x \to \infty} \left(\frac{(x+100)(2x+100)}{(3x+1)(4x+1)} \right)$.

[5] 3. Find $\dfrac{d}{dx}\left(\dfrac{\cos 4x}{\sqrt{x-x^4}}\right)$.

[5] 4. Find $\dfrac{d}{dx}\left(e^{7x}\sin(\ln x)\right)$.

[6] 5. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\left(\sqrt{4x+3}\right)$.

[6] 6. Use implicit differentiation to find $\dfrac{dy}{dx}$ where $x\tan y + y\tan x = 10$.

[15] 7. For the function $f(x) = \dfrac{x^2}{x+1}$, you are given that

$$f'(x) = \frac{x^2 + 2x}{(x+1)^2} \quad \text{and} \quad f''(x) = \frac{2}{(x+1)^3}.$$

(a) Find the domain of $f(x)$.

(b) Find the critical points, and determine whether each critical point is a local maximum, local minimum, or neither.

(c) Find the absolute maximum and absolute minimum of $f(x)$ for x in the interval $[-\frac{1}{2}, 3]$.

(d) Find the intervals where $f(x)$ is concave up and where it is concave down.

(e) Find $\lim_{x \to -1+} f(x)$ and $\lim_{x \to -1-} f(x)$.

[6] 8. Find constants a and b so that the function $f(x) = \begin{cases} 10x - 7 & \text{if } x \leq b \\ x^2 + a & \text{if } x > b \end{cases}$ is both continuous and differentiable at $x = b$.

[6] 9. Prove using the definition of derivative that $\dfrac{d}{dx}(\sin x) = \cos x$. You may use the addition formula $\sin(a+b) = \sin a \cos b + \cos a \sin b$ and the limits $\lim\limits_{h\to 0}\left(\dfrac{\sin h}{h}\right) = 1$ and $\lim\limits_{h\to 0}\left(\dfrac{\cos h - 1}{h}\right) = 0$.

[5] 10. Find and simplify $\int_0^1 (e^x + 2x + 2)\,dx$.

[5] 11. Find and simplify $\int \frac{(2\ln x + 1)^3}{x}\,dx$.

[6] 12. Do **ONE** of the following two problems:

(a) A rectangle is expanding so that its horizontal side is increasing at a rate of 3 cm per minute and its vertical side is increasing at a rate of 4 cm per minute. At some instant the horizontal side is 2 cm long and the vertical side is 6 cm long. At what rate is the area of the rectangle increasing at this instant?

(b) Find all points on the curve $y = x^2 - \frac{9}{2}$ which are closest to the origin $(0, 0)$.

$\overline{[75]}$