

THE UNIVERSITY OF CALGARY
MATHEMATICS 249 L03
FINAL EXAMINATION, WINTER 2010
TIME: 2 HOURS

NAME _____ ID _____ Section _____

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12	
Total (max. 75)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [75]. THIS EXAM HAS 8 PAGES INCLUDING THIS ONE.

[5] 1. Find $\lim_{x \rightarrow 5} \left(\frac{2x - 10}{\sqrt{4 + x} - 3} \right)$. Do not use l'Hôpital's Rule.

[5] 2. Find $\lim_{x \rightarrow 0^+} \left(\frac{e^{x^2} - 1}{x^3} \right)$.

[5] 3. Find $\frac{d}{dx} \left(\frac{e^{2x}}{3x^2 - \cos 3x} \right)$.

[6] 4. USE THE LIMIT DEFINITION OF DERIVATIVE to find $\frac{d}{dx} \left(\frac{1}{x^2} \right)$.

[5] 5. Find y' where $y = \ln(x \sec x - 5)$.

[6] 6. Do **ONE** of the following two questions.

(a) Use implicit differentiation to find the *equation* of the tangent line to the curve given by $x^3 + y^4 = (x - y)^5$ at the point $(-1, -1)$ lying on this curve.

(b) Use logarithmic differentiation to find the *equation* of the tangent line to the curve $y = (x^2 + 1)^{\ln x}$ at the point where $x = 1$.

[13] 7. For the function $f(x) = x\sqrt{x+2}$, you are given that

$$f'(x) = \frac{3x+4}{2\sqrt{x+2}} \quad \text{and} \quad f''(x) = \frac{3x+8}{4(x+2)^{3/2}}.$$

(a) Find the domain of $f(x)$. (Use interval notation.)

(b) Find the intervals of increase and decrease of $f(x)$. Use them to find any relative maxima or relative minima of $f(x)$.

(c) Find the absolute maximum and absolute minimum of $f(x)$ for x in the interval $[-2, 2]$.

(d) Find the intervals where $f(x)$ is concave up and where it is concave down.

[6] 8. The function f is defined by $f(x) = \begin{cases} 2x^2 + 5 & \text{if } x \leq k, \\ x^2 + 4x + 1 & \text{if } x > k, \end{cases}$ where k is a constant.

(a) Find all constants k so that the function f is continuous at $x = k$.

(b) For the value(s) of k found in part (a), is f differentiable at $x = k$? Explain.

[7] 9. An object moves on the number line according to the position function $s(t) = 3t - t^3 - 2$.

(a) Find the velocity $v(t)$ and the acceleration $a(t)$ of the object.

(b) Find all times when the object is moving to the left. (Use interval notation.)

[5] 10. Find and simplify $\int_0^3 \left(1 - \frac{x}{3}\right)^5 dx$.

[5] 11. Find and simplify $\int \frac{\cos \sqrt{x}}{\sqrt{x}} dx$.

[7] 12. Suppose you wish to make a rectangular garden of area 200 square metres, so that one side is along an existing wall and the other three sides are fenced as in the diagram. Find the dimensions of the garden that would use the smallest possible total length of fence.

