Math 249, Winter 2004
Worksheet

1. Solve the following inequalities

   (a) $\dfrac{2}{5-x} \geq \dfrac{4}{x}$

   (b) $(x-4)(x+4) > 0$

2. Find the domain and the range of

   (a) $f(x) = \dfrac{x+2}{x-1}$

3. Find the domain of
   $f(x) = \sqrt{\dfrac{2}{x^2+3}}$

4. Graph the function $\sin(\frac{1}{4}x) + 1$

5. Find the following limits

   (a) $\displaystyle\lim_{x\to\infty} \sqrt[3]{\frac{5x+4}{2x-4}}$

   (b) $\displaystyle\lim_{x\to 0^+} \frac{\sin 3x}{1-\cos 6x}$

   (c) $\displaystyle\lim_{h\to 0} \frac{\tan 4h}{\sin 8h}$

   (d) $\displaystyle\lim_{x\to 1^-} \frac{2x^2+3x+1}{|2x^2+3x+1|}$

6. Let $f(x) = \begin{cases} -x^4+3 & if \quad x \leq 2 \\ x^2+9 & if \quad x > 2 \end{cases}$. Is f continuous everywhere? Justify your conclusion.

7. Use intermediate value Theorem to show that the equation $x^3 - 4x + 2 = 0$ has a solution on the interval $[1, 2]$.

8. Find the points of discontinuity if any of the following functions

   (a) $y = \dfrac{x-4}{x^2-16}$

   (b) $y = \dfrac{\sin\theta}{\theta}$

   (c) $y = \sqrt[3]{3x-1}$

9. Find $\dfrac{dy}{dx}$ of the following

   (a) $\dfrac{1}{y} + \dfrac{1}{2x} = 1$

   (b) $y = (\sin(x^2))^{-\frac{1}{2}}$

   (c) $y = \tan(1 + \sin(x^3))$

   (d) $\sin(x^3y^3) = x^2 + x$

10. Two cyclists start moving form the same point. One travels south at 50km/h and the other travels west at 30km/h. At what rate is the distance between the two cyclists increasing two hours later?

Solutions

1. **a.** $(-\infty, 0) \cup \left[\frac{10}{3}, 5\right)$ **b** $(-\infty, -4) \cup (4, \infty)$

2. Domain $\{x \mid x \neq 1\}$, Range $\{y \mid y \neq 1\}$

3. $(-\infty, \infty)$

4.

5. **a.** $\sqrt[3]{\frac{5}{2}}$ **b.** ∞ **c.** $\frac{1}{2}$ **d**. 1

6. No continuous

7.

8. **a.** $x = -4, 4$ **b.** $\theta = 0$, **c.** No points of discontinuity.

9. **a.** $\frac{dy}{dx} = -\frac{y}{2x^2}$ **b.** $\frac{dy}{dx} = -x\left(\sin x^2\right)^{-\frac{3}{2}} \cos(x^2)$.
   c. $\frac{dy}{dx} = \sec^2\left(1 + \sin(x^3)\right)\left(\cos\left(x^3\right)\right)\left(3x^2\right)$
   d. $\frac{dy}{dx} = \frac{2x + 1 - 3\cos(x^3y^3)x^2y^3}{3x^3y^2\cos(x^3y^3)}$

10. $\frac{6800}{\sqrt{13600}} km/h.$