# Math 249, Winter 2004
# PRACTICE PROBLEMS

1. Express the function $f(x)$ in piecewise form with out using absolute values $f(x) = 3|x-2| - |x+1|$

2. Find or simplify $\dfrac{f(x+h)-f(x)}{h}$ where $h \neq 0$ and $f(x) = 4x^2 - 5x + 7$.

3. Find the domain and the range of the function $g(x)$.

   (a) $g(x) = \sqrt{x^2 - x - 2}$

4. Find the value of the constant $k$, if possible, that will make the function continuous every where.

$$f(x) = \begin{cases} kx^2 & x \leq 2 \\ 2x + k & x > 2 \end{cases}$$

5. State the following Theorems:

   (a) Intermediate-Value Theorem
   (b) Mean-Value Theorem
   (c) Fundamental Theorem of Calculus.

6. What is the definition of the derivative?

7. Show that $f(x) = 5 - x - x^2$ has at least one solution on the interval $[-4, 0]$.

8. Find the following limits

   (a) $\lim\limits_{x\to\infty} \dfrac{3x+4}{\sqrt{2x^2-5}}$

   (b) $\lim\limits_{x\to 0} \dfrac{3x^2}{1-\cos^2(\frac{1}{2}x)}$

   (c) $\lim\limits_{h\to\frac{\pi}{2}} \dfrac{1-\sin h}{\frac{1}{2}\pi - h}$ (Hint use $t = \frac{\pi}{2} - h$)

9. Find $\dfrac{dy}{dx}$ by implicit differentiation.

$$x \sin y + y \cos x = 1$$

10. Find $\dfrac{dy}{dx}$ for the following:

    (a) $y = \sqrt{3}^{x \sin x}$

    (b) $y = (\tan x)^{\ln x}$

11. At which points of the curve $xy = (1 - x - y)^2$ the tangent line is parallel to the $x-$axis?

12. Use local linear approximation to estimate $\cos(46°)$.

13. Solve for $x$ the following equation $\ln\left(\frac{1}{x^2}\right) + \ln\left(4x^6\right) = \ln 4$

14. Find the critical points of the function $f(x) = x^3 - 3x^2 + 3$ and determine if they are maximum, minimum or none.

15. Find the intervals where the function $f(x) = xe^{2x}$ is increasing and decreasing.

16. Find the intervals where the function $f(x) = x^2 \ln x$ is concave up and concave down and determine the inflection points.

17. A rancher has 200feet of fencing with which he wants to enclose two adjacent regular corrals. What dimensions should be used so that the enclose area will be a maximum?

18. Solve the following integrals:

    (a) $\int \sin(2x)\cos(2x)dx$

    (b) $\int \frac{\csc^2 x}{\cot^3 x}dx$

    (c) $\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right)dx$

    (d) $\int_1^3 \frac{e^{\frac{3}{x}}}{x^2}dx$

    (e) $\frac{d}{dx}\int_x^0 \frac{t}{\cos t}dt$

*Solutions*

1. $f(x) = \begin{cases} -2x - 7 & \text{if} \quad x < -1 \\ -4x + 7 & \text{if} \quad -1 \leq x < 2 \\ 2x - 7 & \text{if} \quad x \geq 2 \end{cases}$

2. $8x + 4h - 5$

3. Domain $(-\infty, -1] \cup [2, \infty)$
   Range $y \geq 0$.

4. $k = 4/3$

5.

6.

7.

8. (a) $\frac{3}{\sqrt{2}}$

(b) 12

(c) 0

9. $\dfrac{dy}{dx} = \dfrac{y\sin x - \sin y}{x\cos y + \cos x}$

10. (a) $\dfrac{dy}{dx} = \left(\ln\sqrt{3}\left[\sin x + x\cos x\right]\right)\ln\sqrt{3}^{x\sin x}$

    (b) $\dfrac{dy}{dx} = \left(\dfrac{\ln(\tan x)}{x} + \dfrac{\ln x}{\tan x}\sec^2 x\right)\ln(\tan x)^{\ln x}$

11. $y = 2 - 2x$

12. $\cos(46°) \approx 1/\sqrt{2}\left(1 - \frac{\pi}{180}\right)$.

13. $x = \pm 1$

14. 0 is a maximum and 2 is a minimum.

15. $f(x)$ is increasing on $\left(-\frac{1}{2}, \infty\right)$ and decreasnig on $\left(-\infty, -\frac{1}{2}\right)$

16. $f(x)$ is concave up on $\left(e^{-\frac{3}{2}}, \infty\right)$ and concave down on $\left(-\infty, e^{-\frac{3}{2}}\right)$. $x = e^{-\frac{3}{2}}$ is an inflection point.

17. Dimensions: 50feet by 100/3feet.

18. (a) $\dfrac{\sin^2(2x)}{4} + c$

    (b) $\dfrac{1}{\cot^2 x} + c$

    (c) $2\sqrt{3} - 2$

    (d) $-\frac{1}{3}\left(e + e^3\right)$

    (e) $-\dfrac{x}{\cos x}$