LABS 3, Feb. 16, 2005

1 True / False

If $f(a) < 0$ and $f(b) > 0$ (with $a < b$) then there is at least one x in $(a, b]$ with $f(x) = 0$.

Not true in general; true if f is continuous — F

2(a) Let $y = \frac{1}{x^2}$. The average rate of change of y with respect to x on $[3, 4]$ is $\frac{1}{16} - \frac{1}{9}$

$$\frac{\frac{1}{x^2}\Big|_4 - \frac{1}{x^2}\Big|_3}{4-3} = \frac{\frac{1}{16} - \frac{1}{9}}{1}$$

(b) The instantaneous rate of change of y at $x = 3$ is $-\frac{2}{27}$

$$\frac{dy}{dx}\Big|_{x=3} = \frac{d}{dx}\left(\frac{1}{x^2}\right)\Big|_{x=3} = \frac{d}{dx}\left(x^{-2}\right)\Big|_{x=3}$$

3 If $g(x) = \frac{1}{x-2}$, $x \neq 2$ and $(f \circ g)(x) = x$ then

$f(x) =$ { how do we get x from $\frac{1}{x-2}$? undo, and add 2 } $\frac{1}{x} + 2$

4 $\lim_{h \to 0} \frac{\sin\left(\frac{\pi}{4} + h\right) - \sin\left(\frac{\pi}{4}\right)}{h} = \frac{d}{dx}(\sin x)\Big|_{x=\pi/4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

5 $\lim_{x \to 0} \left(\frac{x \sin x}{\cos x - 1}\right) = -2$

Rationalize denominator; use $\sin^2 x + \cos^2 x = 1$

6 The equation of the tangent line to the curve $y = \sin x$ at $x = \frac{\pi}{4}$ is: $x = \pi/4 \Rightarrow y = \frac{1}{\sqrt{2}}$ $y - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(x - \frac{\pi}{4}\right)$

7 $\frac{d}{dx}\left(\frac{x^2}{x^4+1}\right) = \frac{(2x)(x^4+1) - x^2(4x^3)}{(x^4+1)^2}$ $\frac{d}{dx}\left(\sec(2x)\right) = \sec(2x)\tan(2x)\,2$

8 For which value of x is the tangent line to the curve $y = x^2 - x$ parallel to the line $2x - y + 4 = 0$

$\frac{dy}{dx} = \frac{d}{dx}(x^2 - x) = 2x - 1 =$ slope of tangent line $2x - 1 = 2$, $x = \frac{3}{2}$

$= 2$, since slope of line $2x - y + 4 = 0$ is 2