90 Minute Exam (Multiple Choice) 249

1. Find $\lim_{x\to 2}\left(\dfrac{x^3-8}{x-2}\right)$.

   (A) 12
   (B) 4
   (C) ∞
   (D) does not exist
   (E) 8

2. Find $\lim_{t\to 0}\left(\dfrac{\sqrt{t^2+4}-2}{t}\right)$.

   (A) -1
   (B) does not exist
   (C) 1
   (D) 2
   (E) 0

3. Find $\lim_{x\to\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2-3x}\right)$.

   (A) ∞
   (B) -3
   (C) $-2\sqrt{3}$
   (D) 3
   (E) $2\sqrt{3}$

4. Find $\lim_{x\to\infty}\dfrac{5x^2-2x-3}{(2x+3)(3x+2)}$.

   (A) $-1/2$
   (B) ∞
   (C) $-\infty$
   (D) $5/6$
   (E) $5/13$

5. Find $\lim_{x\to-\infty}\left(\dfrac{3x+7}{\sqrt{4x^2+5}}\right)$.

   (A) $7/\sqrt{5}$
   (B) $-3/\sqrt{5}$
   (C) 0
   (D) $3/2$
   (E) $-3/2$

6. The derivative of $4x^3 + 3x^2 + 2x + 1$ is

(A) $x^4 + x^3 + x^2 + x$
(B) $4x^2 + 3x + 2$
(C) $(x^5 - 1)/(x - 1)$
(D) $x^4 + x^3 + x^2 + x + C$
(E) $12x^2 + 6x + 2$

7. The derivative of $\dfrac{x^2}{x^4 + 1}$ is

(A) $\dfrac{1}{2x^2}$
(B) $\dfrac{6x^5 + 2x}{(x^4 + 1)^2}$
(C) $\dfrac{2x^5 - 2x}{(x^4 + 1)^2}$
(D) $\dfrac{2x - 4x^3}{(x^4 + 1)}$
(E) $\dfrac{2x - 2x^5}{(x^4 + 1)^2}$

8. The derivative of $x^2 \tan(3x)$ is

(A) $2x \tan(3x) + 3x^2 \tan(3x) \sec(3x)$
(B) $6x \sec^2(3x)$
(C) $x^2 \cot(3x)$
(D) $2x \tan(3x) + 3x^2 \sec^2(3x)$
(E) $6x \tan(3x)$

9. The derivative of $\ln(x^3)$ is

(A) $3/x$
(B) $x/\ln(x)$
(C) $3x^2 \ln(x^3)$
(D) $3x^2/\ln(x^3)$
(E) $3x^2/\ln(x)$

10. The derivative of $y = x^{\cos(x)}$ is

(A) $(\cos(x)/x - \sin(x)\ln(x))x^{\cos(x)}$
(B) $e^x x^{\cos(x)}$
(C) $\ln(x)x^{\cos(x)}$
(D) $-\sin(x)x^{\cos(x)-1}$
(E) $(\sin(x)/x - \ln(x))x^{\cos(x)}$

11. The antiderivative $\int \frac{1}{x^2} + x^2\,dx$ is

(A) $\ln(x^2)/2x + x^3/3 + C$
(B) $x^2/x^3 + x^3/x + C$
(C) $-2/x^3 + 2x + C$
(D) $-1/x + x^3/3 + C$
(E) $\arccos(x^2) + C$

12. The antiderivative $\int \frac{1}{x} + xe^{x^2}\,dx$ is

(A) $x^0 + e^{x^2} + C$
(B) $\ln|x| + e^{x^2}/2 + C$
(C) $-1/x^2 + 2x^2e^{x^2} + C$
(D) $\ln|x| + \ln(e^{x^2}) + C$
(E) $-1/x^2 + (2x^2+1)e^{x^2} + C$

13. The antiderivative $\int \sin(\frac{x}{3})\,dx$ is

(A) $3\cos(\frac{x}{3}) + C$
(B) $\frac{1}{2}\sin^2(\frac{x}{3}) + C$
(C) $-3\cos(\frac{x}{3}) + C$
(D) $-\cos(\frac{x}{3}) + C$
(E) $\cos(\frac{x}{3}) + C$

14. The antiderivative $\int \tan^2 x\,dx$ is

(A) $\tan x - x + C$
(B) $\ln(\cos^2 x) + C$
(C) $\sec x + x + C$
(D) $\tan x + C$
(E) $\sec x + C$

15. The antiderivative $\int \frac{1}{\sqrt{4-x^2}}\,dx$ is

(A) $4\arcsin(x/4) + C$
(B) $\arcsin(x/2) + C$
(C) $-\frac{\sqrt{4-x^2}}{2x} + C$
(D) $2\arccos(x/2) + C$
(E) $\frac{x}{(4-x^2)^{3/2}} + C$

16. The statement $\lim_{x \to a} f(x) = L$ means

(A) $f(a) = L$
(B) if $x < a$ then $f(x) < L$, and if $x > a$ then $f(x) > L$
(C) the function f is continuous at $x = a$
(D) For x close to a, $f(x)$ will be close to L
(E) for x close to a, but $x \neq a$, $f(x)$ will be close to L

17. Which function is not continuous on its domain?

(A) $x/|x|$
(B) $\ln x$
(C) $|x|$
(D) $1/(x^2 - 1)$
(E) $\lfloor x \rfloor$

18. If $f(x) = x^3$, which of the following limits equals $f'(2)$?

(A) $\lim_{x \to 2} \dfrac{x^3 - 8}{x - 2}$
(B) $\lim_{h \to 0} \dfrac{(2 - h)^3 + h^3}{h}$
(C) $\lim_{h \to 0} \dfrac{3h + 3h^2 + h3}{h}$
(D) $\lim_{x \to 2} \dfrac{(2 + h)^3 - h^3}{h}$
(E) $\lim_{h \to 0} \dfrac{3x^2h + 3xh^2 + h^3}{h}$

19. The function $f(x) = \sqrt{x^2 - 1}$ has natural domain:

(A) $[-1, 1]$
(B) $[1, \infty)$
(C) $(-\infty, -1] \cup [1, \infty)$
(D) $(-\infty, \infty)$
(E) $[0, 1]$

20. The line $y = 3x + 1$ is tangent to some curve at point $(\frac{1}{3}, 2)$. The normal line, at the same point, is

(A) $y = -\frac{x}{3} + \frac{19}{9}$
(B) $y = 3x - 1$
(C) $y - 2 = \frac{1}{3}(x - \frac{1}{3})$
(D) $y = -\frac{x}{3} + 1$
(E) None of these.

21. The value of $\tan(5\pi/4)$ is

(A) $\sqrt{2}/2$
(B) $-\sqrt{2}/2$
(C) 1
(D) $\sqrt{3}$
(E) -1

22. How many solutions are there to the equation $4(2^x)^x = 8^x$.

(A) 1
(B) 3
(C) 0
(D) 2
(E) more than 3

23. Use the laws of logs to simplify $\log_{15} 75 + \log_{15} 3$.

(A) 2
(B) $\log_{15} 78$
(C) $5 + 1/5$
(D) $\log(5) + \log(1/5)$
(E) $\log_{30} 225$

24. Evaluate $\lim_{x \to -\infty} \tanh(x)$.

(A) $-\infty$
(B) $+1$
(C) $+\infty$
(D) -1
(E) 0

25. Evaluate $\sinh(\ln 3)$.

(A) $(e^3 - e^{-3})/2$
(B) $5/3$
(C) $4/3$
(D) $\cosh(-\ln 3)$
(E) $(\ln(3) + \ln(1/3))/2$

26. A car travels $7t + 2t^2$ metres in t seconds.
Let V_1 be the instantaneous velocity at $t = 2$ secs,
V_2 the average velocity in the interval $2 \leq t \leq 2.5$ sec,
V_3 the average velocity in the interval $2 \leq t \leq 3$ sec.

(A) $V_1 = 15m/s, V_2 = 16.5m/s, V_3 = 17m/s$
(B) $V_1 = 15m/s, V_2 = 16m/s, V_3 = 17m/s$
(C) $V_1 = 16m/s, V_2 = 17m/s, V_3 = 15m/s$
(D) $V_1 = 22m/s, V_2 = 17m/s, V_3 = 16m/s$
(E) $V_1 = 22m/s, V_2 = 15m/s, V_3 = 14m/s$

27. Imagine stretching a long string around the earth's equator, where the radius of the earth is R metres. Cut the string, and raise it everywhere so it is one metre above the earth everywhere. How much extra string do you need to rejoin the cut ends?

(A) 2π metres
(B) R/π metres
(C) $(R/\pi)^2$ metres
(D) πR metres
(E) π metres

28. For the function $y = 6\sqrt[3]{x}$, if x changes from 1000 to 1001, the approximate change in y is

(A) $dy = 2x^{-2/3}$
(B) $dy = 6(\sqrt[3]{1001} - 10)$
(C) $dy = .01861$
(D) $dy = \Delta y$
(E) $dy = 1/50$

29. If $y = x^{4/3}$ and x increases by 3%, then y will change by approximately

(A) 4% (B) 2% (C) 0% (D) 1% (E) 3%

30. Suppose f, g are differentiable functions, with
$f(2) = 3, f'(2) = 6, f'(-1) = 5$ and
$g(2) = -1, g'(2) = 7, g'(-1) = 0$. Then $(f \circ g)'(2)$ is

(A) 42 (B) does not exist (C) 0 (D) 35 (E) -5

31. The tangent line to curve $y = \cos(2x)$ at point $(\frac{\pi}{6}, \frac{1}{2})$ is

(A) $y = -\sqrt{3}x + \frac{1}{2} + \frac{\sqrt{3}\pi}{6}$
(B) $y = \sqrt{2}x + \frac{1}{2} - \frac{\sqrt{2}\pi}{6}$
(C) $y = \sqrt{3}x + \frac{1}{2} - \frac{\sqrt{3}\pi}{6}$
(D) $y = \frac{1}{2} - 2\sin(2x) \cdot (x - \frac{\pi}{6})$
(E) $y = -\sqrt{2}x + \frac{1}{2} + \frac{\sqrt{2}\pi}{6}$

32. Given the curve $x^2y^3 = 4x - y$ which passes through the point $P = (3, 1)$, the slope at P equals

(A) $(4 - 2xy^3)/(3x^2y^2 + 1)$
(B) $3/(3x^2y^2 + 1)$
(C) $-25/14$
(D) $4/(3x^2y^2 + 1)$
(E) $-1/14$

33. The curve in the previous question also passes through $(0, 0)$. The Mean Value Theorem says

(A) for some x between 0 and 3, $\frac{dy}{dx} = -3$
(B) for some x between 0 and 1, $\frac{dy}{dx} = \frac{1}{3}$
(C) for some x near 0, $\frac{dy}{dx} = 3$
(D) for some y between 0 and 1, $\frac{dy}{dx} = 3$
(E) for some x between 0 and 3, $\frac{dy}{dx} = \frac{1}{3}$

34. Given the initial value problem $y' = x^{1/3}, y(1) = \frac{11}{4}$, then the solution $y = \frac{3}{4}x^{4/3} + C$ has constant

(A) $C = 1$
(B) $C = -2$
(C) $C = 2$
(D) $C = 0$
(E) $C = 11/4$

35. The inverse to function $f(x) = \dfrac{x}{x+1}$ is

(A) $f^{-1}(x) = \frac{x+1}{x}$
(B) $f^{-1}(x) = \frac{1}{(x+1)^2}$
(C) $f^{-1}(y) = \frac{x}{x+1}$
(D) $f^{-1}(x) = \frac{x}{1-x}$
(E) $f^{-1}(y) = \frac{y}{y+1}$

36. Suppose the function $f(x) = \dfrac{4x^3}{x^2+1}$ has inverse g. Find $g'(2)$.

(A) 25/112
(B) 1/4
(C) 4
(D) 1
(E) 112/25

37. Compute $\lim_{x\to\infty} x^{-4}e^x$

(A) 0 (B) 1 (C) e (D) 4 (E) ∞

38. Determine on what interval(s) the function $f(x) = xe^{-x}$ is increasing.

(A) $[1, \infty)$
(B) $(-\infty, 1]$
(C) $[2, \infty]$
(D) $[-\infty, 0] \cup [1, \infty)$
(E) $[1, 2]$

39. Plutonium is a radioactive element with a half-life of approximately 24,000 years. If you have 100 grams of plutonium today, how much remains after 36,000 years?

(A) $50/\sqrt{2}$ grams
(B) $100e^{(3/2)\ln 2}$ grams
(C) 25 grams
(D) $50\sqrt{2}$ grams
(E) $100e^{-36{,}000}$ grams

40. With $u = \sqrt{(x+1)(x^2+1)(x^3+1)}$, use logarithmic differentiation to find $u'(1)$.

(A) $3\sqrt{2}$
(B) $\ln(2\sqrt{2})$
(C) 3/2
(D) $\ln(6)$
(E) $2\sqrt{2}$