Mathematics 249, Quiz 2, Week of Feb. 7, 2005

NAME (PRINT) I.D.

SIGNATURE

1 Show that the equation $x^3 = x - 4$ has at least one solution in the interval $[-2, 0]$

(2) Let $f(x) = x^3 - (x-4) = x^3 - x + 4$

To show that $x^3 = x - 4$ is equivalent to showing that $f(x)$ is zero.

Now $f(x)$, being the difference of two continuous functions, is continuous.

Also, $f(-2) = (-2)^3 - (-2) + 4 = -8 + 6 = -2 < 0$ and $f(0) = 4 > 0$. Since f changes sign between -2 and 0, there is at least one value of x between ~~-2 and 0 with $f(x)=0$~~ using the IMT

2 Let $f(x) = \dfrac{4x^2 + x + 10}{x^2 - 1}$

(2)

(a) Find the horizontal asymptote of $f(x)$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\left(4 + \frac{1}{x} + \frac{10}{x^2}\right)}{\left(1 - \frac{1}{x^2}\right)} = 4 = \lim_{x\to\infty} y$$

Thus $y = 4$ is the H.A.

(b) Find the vertical asymptotes of $f(x)$

$$x^2 - 1 = 0 \Rightarrow x^2 - 1^2 = 0$$

$$\Rightarrow (x-1)(x+1) = 0 \Rightarrow x = 1 \text{ OR } x = -1$$

The vertical asymptotes are $x = 1$ and $x = -1$

3 True or false:

(2) (a) If $f(x)$ is continuous at $x = a$ then $\lim_{x\to a} f(x) = f(a)$ T

(b) If $f(x)$, $g(x)$ are continuous at $x = a$, then $f(x) + g(x)$ is continuous at $x = a$. T

4 Let $f(x) = \begin{cases} \dfrac{1-x^3}{1-x}, & x \neq 1 \\ k, & x = 1 \end{cases}$

(2)

Which value of k ensures that $f(x)$ is continuous? Show your work

For continuity we must have

$$\lim_{x \to 1} (f(x)) = f(x)\Big|_{x=1} = f(1) = k$$

$$\text{So} \quad k = \lim_{x \to 1} \left(\frac{1-x^3}{1-x} \right) = \lim_{x \to 1} \frac{\cancel{(1-x)}(1+x+x^2)}{\cancel{(1-x)}}$$

$$= \lim_{x \to 1} (1+x+x^2) = 3$$

5 Let $f(x)$, $g(x)$ be continuous functions with $f(1) = 3$ and such that $\lim_{x \to 1} (2f(x) + g(x)) = 7$

(2)

Find (a) $\lim_{x \to 1} (g(x))$ (b) $g(1)$. Show your work.

We are given that $\lim_{x \to 1} (2f(x) + g(x)) = 7$

So, $\lim_{x \to 1} (2f(x)) + \lim_{x \to 1} (g(x)) = 7$

Thus $2 \lim_{x \to 1} (f(x)) + \lim_{x \to 1} (g(x)) = 2f(1) + g(1) = 7$

Here we use that f, g are continuous at $x = 1$.

Then $2(3) + g(1) = 7$, $g(1) = 1 = \lim_{x \to 1} g(x)$

6 Evaluate the following limits

(5)

(a) $\lim_{x \to 1} \left(\dfrac{\sqrt{4+x}}{1+x^2} \right)$ $\dfrac{\sqrt{5}}{2}$ (b) $\lim_{x \to 0} \left(\dfrac{\sin(5x)}{2x} \right)$ $\dfrac{5}{2}$

(c) $\lim_{x \to 0^+} \left(\dfrac{1}{x} - \dfrac{1}{x^2} \right)$ $-\infty$ (d) $\lim_{x \to 0} \left(\dfrac{|2x+3| - 3}{x} \right)$ 2

(e) $\lim_{x \to 3} \left(\dfrac{\sqrt{x^2+2} - \sqrt{11}}{x-3} \right)$ $\dfrac{3}{\sqrt{11}}$