

THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
MATH 249 L01/L02 - Fall 2004

Time: 2 hours

I.D. NUMBER	SURNAME	OTHER NAMES

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. A student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.

EXAMINATION RULES

1. Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
2. No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
3. All enquiries and requests must be addressed to supervisors only.
4. Candidates are strictly cautioned against:
 - (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
 - (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
 - (c) making use of calculators and/or portable computing machines not authorized by the instructor;
 - (d) leaving answer papers exposed to view;
 - (e) attempting to read other students' examination papers.

The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.

5. Candidates are requested to write on both sides of the page, unless the examiner has asked that the left half page be reserved for rough drafts or calculations.
6. Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
7. Candidates are cautioned against writing in their answer books any matter extraneous to the actual answering of the question set.
8. The candidate is to write his/her name on each answer book as directed and is to number each book.
9. A candidate must report to a supervisor before leaving the examination room.
10. Answer books must be handed to the supervisor-in-charge promptly when the signal is given. Failure to comply with this regulation will be cause for rejection of an answer paper.
11. If a student becomes ill or receives word of domestic affliction during the course of an examination, he/she should report at once to the Supervisor, hand in the unfinished paper and request that it be cancelled. Thereafter, if illness is the cause, the student must go directly to University Health Services so that any subsequent application for a deferred examination may be supported by a medical certificate. An application for Deferred Final Examinations must be submitted to the Registrar by the date specified in the University Calendar.
Should a student write an examination, hand in the paper for marking, and later report extenuating circumstances to support a request for cancellation of the paper and for another examination, such request will be denied.

12. SMOKING DURING EXAMINATIONS IS STRICTLY PROHIBITED.

NOTE: A basic function calculator is allowed.

Important instructions:**Put your answers in the answer sheet provided.**

1. Find the domain of the function $f(x) = \frac{\sqrt{x+2}}{x} - \frac{1}{x+4}$.
- (a) $(-2, 0) \cup (0, \infty)$
 - (b) $[-2, 0) \cup (0, \infty)$
 - (c) $(-4, -2) \cup (-2, 0) \cup (0, \infty)$
 - (d) $[-4, -2) \cup (-2, 0) \cup (0, \infty)$
 - (e) None of the above.
2. If $2\sin^2 x - 3\sin x + 1 = 0$, where $-\pi \leq x \leq \pi$, then $x =$
- (a) $0, \pi/6, 5\pi/6, \pi$.
 - (b) $0, \pi/3, 2\pi/3, \pi$.
 - (c) $\pi/6, \pi/2, 5\pi/6$.
 - (d) $\pi/3, \pi/2, 2\pi/3$.
 - (e) None of the above.
3. If $f(x) = 4x - 5$, $g(x) = \frac{1}{x+1}$, and $h(x) = \sqrt{x}$, then $(f \circ g \circ h)(x) =$
- (a) $\frac{(4x-5)\sqrt{x}}{x+1}$.
 - (b) $\frac{4}{\sqrt{x}+1} - 5$.
 - (c) $\frac{1}{\sqrt{4x-5}+1}$.
 - (d) $4\sqrt{\frac{1}{x+1}} - 5$.
 - (e) None of the above.

4. Find $\lim_{x \rightarrow -\infty} \frac{4x}{3x - \sqrt{x^2 + 4}}$

- (a) 1
- (b) 2
- (c) -1
- (d) -2
- (e) None of the above.

5. Suppose ε is a small positive real number. How close to 5 must we hold x to be sure that x^2 lies within ε units of 25?

- (a) We must hold x within $\sqrt{\varepsilon}$ units of 5.
- (b) We must hold x within $\varepsilon/2$ units of 5.
- (c) We must hold x within $\sqrt{25 + \varepsilon} - 5$ units of 5.
- (d) We must hold x within $5 - \sqrt{25 - \varepsilon}$ units of 5.
- (e) None of the above.

6. Which of the following limits is an indeterminate form?

- (a) $\lim_{x \rightarrow \pi^+} \frac{\tan x}{x}$
- (b) $\lim_{x \rightarrow 0^-} \csc x \cot x$
- (c) $\lim_{x \rightarrow \infty} \left(\frac{x}{x+1} \right)^{1/x}$
- (d) $\lim_{x \rightarrow \infty} \left(\frac{1}{e^x} - \frac{1}{x} \right)$
- (e) None of the above.

7. Let $f(x) = \begin{cases} c^2 x^3, & \text{if } x < 1, \\ 5cx + 6, & \text{if } 1 \leq x. \end{cases}$ Find the value(s) of c such that f is continuous at 1.

- (a) 2 and 3
- (b) -2 and -3
- (c) 1 and -6
- (d) -1 and 6
- (e) None of the above.

8. Let $y = x^3 e^{-x}$. Then $y'' =$

- (a) $x(x^2 + 6)e^{-x}$
- (b) $x(-x^2 + 6)e^{-x}$
- (c) $x(x^2 - 6x + 6)e^{-x}$
- (d) $x(-x^2 - 6x + 6)e^{-x}$
- (e) None of the above.

9. Find the derivative of $y = \frac{x-3}{x^2+2}$

- (a) $\frac{-x^2 + 6x + 2}{(x^2 + 2)^2}$
- (b) $\frac{x^2 - 6x - 2}{(x^2 + 2)^2}$
- (c) $\frac{3x^2 - 6x + 2}{(x^2 + 2)^2}$
- (d) $\frac{-x^2 - 1}{(x^2 + 2)^2}$
- (e) None of the above.

10. Differentiate $g(x) = e^x (3 \sin x + 5 \cos x)$

- (a) $e^x (2 \sin x - 8 \cos x)$
- (b) $e^x (-2 \sin x + 8 \cos x)$
- (c) $e^x (2 \sin x - 2 \cos x)$
- (d) $e^x (-2 \sin x + 2 \cos x)$
- (e) None of the above.

11. If $f(x) = (2x + \cos x) \sqrt{x^2 + 1}$, find $f'(0)$.

- (a) -1
- (b) 0
- (c) 1
- (d) 2
- (e) None of the above.

12. Suppose that y is a differentiable function of x that satisfies $e^x \cos y + y^3 \sin x = 2$.

Find $\frac{dy}{dx}$.

- (a) $\frac{e^x \cos y + y^3 \cos x}{e^x \sin y - 3y^2 \sin x}$
- (b) $\frac{e^x \cos y + y^3 \cos x}{3y^2 \sin x - e^x \sin y}$
- (c) $\frac{e^x \cos y - y^3 \cos x}{e^x \sin y + 3y^2 \sin x}$
- (d) $\frac{e^x \cos y + y^3 \cos x}{e^x \sin y + 3y^2 \sin x}$
- (e) None of the above.

13. If $f(x) = (3x + 4)^x \sec x$, find $f'(x)$.

- (a) $(3x + 4)^x \sec x \left[\ln(3x + 4) + \frac{3}{3x + 4} + \tan x \right]$
- (b) $(3x + 4)^x \sec x \left[\ln(3x + 4) + \frac{x}{3x + 4} + \tan x \right]$
- (c) $(3x + 4)^x \sec x \left[\ln(3x + 4) + \frac{3x}{3x + 4} + \tan x \right]$
- (d) $(3x + 4)^x \sec x \left[\frac{3}{3x + 4} + \tan x \right]$
- (e) None of the above.

14. Find the equation of the tangent line to the graph of $y = \sec^2(\pi x)$ at the point $\left(\frac{1}{4}, 2\right)$.

(a) $y - 2 = 2\sqrt{2}\left(x - \frac{1}{4}\right)$.

(b) $y - 2 = 2\sqrt{2}\pi\left(x - \frac{1}{4}\right)$.

(c) $y - 2 = 4\left(x - \frac{1}{4}\right)$.

(d) $y - 2 = 4\pi\left(x - \frac{1}{4}\right)$.

(e) None of the above.

15. A paper cup has the shape of a cone with height 5 cm and (top) radius 2 cm. If water is poured into the cup at a rate of $1 \text{ cm}^3/\text{s}$, how fast is the water level rising when the water is 2 cm deep?

(a) $\frac{3}{4\pi} \text{ cm/s}$

(b) $\frac{5}{8\pi} \text{ cm/s}$

(c) $\frac{25}{4\pi} \text{ cm/s}$

(d) $\frac{25}{16\pi} \text{ cm/s}$

(e) None of the above.

16. Suppose that $x = 2$ is a critical point of $f(x) = x^3 e^{-kx}$. Find the value of k .

- (a) 3.
- (b) -3 .
- (c) $3/2$.
- (d) $-3/2$.
- (e) None of the above.

17. Let $f'(x) = x^{-4/3} - x^{-1/3}$. Find the largest open intervals on which f is increasing.

- (a) $(0, 1)$ only
- (b) $(0, 1)$ and $(1, \infty)$
- (c) $(-\infty, 0)$ and $(1, \infty)$
- (d) $(-\infty, 0)$ and $(0, 1)$
- (e) None of the above.

18. If $f'(x) = x(\ln|x|)^2$, then find the largest open interval(s) on which f is concave downward.
- (a) $(0, 1)$ and $(1, \infty)$
 - (b) $(-1, -e^{-2})$ and $(e^{-2}, 1)$
 - (c) $(0, e^{-2})$ and $(1, \infty)$ only
 - (d) $(-\infty, -1)$, $(-e^{-2}, 0)$, $(0, e^{-2})$, and $(1, \infty)$
 - (e) None of the above.

19. Given

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	3	4	5	4
3	4	2	0	3
4	2	3	-1	-2

Find $\left(\frac{f}{g}\right)'(2)$.

- (a) $1/2$
- (b) $3/4$
- (c) $5/4$
- (d) 2
- (e) None of the above.

20. Find the slant asymptote of the function $f(x) = \frac{x^2 + 5x + 2}{x - 3}$.

- (a) $y = x + 8$.
- (b) $y = x - 8$.
- (c) $y = x + 2$.
- (d) $y = x - 2$.
- (e) None of the above.

21. The slope of the tangent line to the graph of $y = f(x)$ at the point $(a, f(a))$ is given by $m = \frac{1}{a} - \frac{1}{a^2}$. If the graph of f passes through the point $(1, 2)$, find $f(x)$.

- (a) $f(x) = \ln x + \frac{1}{x} + 1$
- (b) $f(x) = \ln x - \ln(x^2) + 2$
- (c) $f(x) = -\frac{1}{x^2} + \frac{2}{x^3} + 1$
- (d) $f(x) = 1 - \frac{1}{2x}$
- (e) None of the above.

22. The velocity of a moving object at time t is given by $v(t) = \frac{e^t}{e^t + 1}$. Find the position $s(t)$ at time t , if $s(0) = 0$.

- (a) $\frac{e^t}{e^t + 1} - 1$
- (b) $\ln(e^t + 1)$
- (c) $\ln(e^t + 1) - \ln 2$
- (d) $e^t \ln(e^t + 1) - \ln 2$
- (e) None of the above.

23. If $f(x) = x^{-2x}$, then $f'(x) =$

- (a) $-2x(x^{-2x-1})$
- (b) $-2x^{-2x}(\ln x - 1)$
- (c) $-2x^{-2x}(\ln x + 1)$
- (d) $-2x^{-2x}\left(\ln x + \frac{1}{x}\right)$
- (e) None of the above.

24. Find the linearization of $f(x) = (5 - 4x)^{1/3}$ at 1.

(a) $L(x) = 1 + \frac{1}{3}(x - 1)$

(b) $L(x) = 1 - \frac{1}{3}(x - 1)$

(c) $L(x) = 1 + \frac{4}{3}(x - 1)$

(d) $L(x) = 1 - \frac{4}{3}(x - 1)$

(e) None of the above.

25. Find $\int \left(\sqrt{t} - \frac{1}{\sqrt{t}} \right) dt$

(a) $\frac{2}{3}t^{3/2} + 2t^{1/2} + C$

(b) $\frac{2}{3}t^{3/2} - 2t^{1/2} + C$

(c) $\frac{2}{3}t^{3/2} - \frac{1}{\frac{2}{3}t^{3/2}} + C$

(d) $\frac{1}{2}t^{-1/2} - 2t^{1/2} + C$

(e) None of the above.

26. It is easy to verify that $\frac{d}{dx} [x^3 \ln x] = x^2 (3 \ln x + 1)$. Find $\int_1^2 9x^2 \ln x dx$.

- (a) 9
- (b) $24 \ln 2$
- (c) $24 \ln 2 - 7$
- (d) $24 \ln 2 - 9$
- (e) None of the above.

27. The substitution $u = 2x + 3$ transforms $\int \frac{4x - 1}{\sqrt{2x + 3}} dx$ into

- (a) $\int \frac{u - 2}{\sqrt{u}} du$
- (b) $\int \frac{2u - 7}{2\sqrt{u}} du$
- (c) $\int \frac{4u - 1}{\sqrt{u}} du$
- (d) $(4x - 1) \int \frac{1}{2\sqrt{u}} du$
- (e) None of the above.

28. Find $\int \sin(\cos x) \sin x dx$.

- (a) $\cos(-\sin x) \cos x + C$
- (b) $\cos(\cos x) + C$
- (c) $-\cos(\cos x) + C$
- (d) $-\cos(\sin x) + C$
- (e) None of the above.

29. Find $\int \frac{\sec \sqrt{x} \tan \sqrt{x}}{\sqrt{x}} dx$.

- (a) $\sec \sqrt{x} + C$
- (b) $2 \sec \sqrt{x} + C$
- (c) $\sqrt{x} \sec \sqrt{x} + C$
- (d) $2\sqrt{x} \sec \sqrt{x} + C$
- (e) None of the above.

30. $\int \frac{\sin x \cos x}{\sqrt{1 + \sin^2 x}} dx =$

- (a) $\sqrt{1 + \sin^2 x} + C.$
- (b) $2\sqrt{1 + \sin^2 x} + C.$
- (c) $\sin^2 x \sqrt{1 + \sin^2 x} + C.$
- (d) $\frac{1}{2} \sin^2 x \sqrt{1 + \sin^2 x} + C.$
- (e) None of the above.

31. Find $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x + 2} dx.$

- (a) -3
- (b) 3
- (c) $-\frac{1}{2} \ln \frac{3}{2}$
- (d) $\frac{1}{2} \ln \frac{3}{2}$
- (e) None of the above.

32. Find $f''(e)$ if $f(x) = \int_1^x \frac{t}{\ln t + 1} dt$.

- (a) $1/4$
- (b) $1/2$
- (c) 1
- (d) $e/2$
- (e) None of the above.

33. $\int_0^4 |x - 2| dx =$

- (a) 0
- (b) 2
- (c) 4
- (d) 6
- (e) None of the above.

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