LABS 4, Math 249, March 9, 2005

1 The local linear approximation of $f(x) = \sqrt{9+x^2}$

at values of x close to 0 is given by $\underline{\sqrt{9+x^2} \doteq 3}$

$f(x) \doteq f(a) + f'(a)(x-a)$ with

$a = 0$ and $f(x) = \sqrt{9+x^2}$ etc

2 For the function $y = 6\sqrt[3]{x}$, if x changes from 1000 to 1001 then the approximate change in y is

$f(x) \doteq f(a) + f'(a)(x-a)$, $f(x) - f(a) \doteq f'(a)(x-a)$, $x-a = 1$, $a = 1000$

(A) $dy = 2x^{-2/3}$ (B) $dy = 6(\sqrt[3]{1001} - 10)$ (C) $dy = .01861$

(D) $dy = \Delta y$ (E) $dy = \frac{1}{50}$

3 Let $f(x) = \frac{3}{x^2}$, $x < 0$. $f^{-1}(x) = -\sqrt{\frac{3}{x}}$

(a) Find $f^{-1}(x)$, $y = \frac{3}{x^2}$, $x^2 = \frac{3}{y}$, $x = \pm\sqrt{\frac{3}{y}}$ etc

(b) Find the domain of $f^{-1}(x)$ = Range of f = $(0, \infty)$

4 Find x if $\ln\left(\frac{5}{x}\right) + \ln(2x^3) = \ln(40)$

$\ln\left(\frac{5}{x}\right)(2x^3) = \ln 40$, $\ln(10x^2) = \ln 40$, $x^2 = 4$, $x = 2$

5 Let $f(x) = x^4 + x^3 + 1$ for $0 \le x \le 2$

(a) Show that f^{-1} exists (b) Find $f^{-1}(3)$

(a) f' is > 0 on $[0, 2]$ so f incr., f^{-1} exists. (b) $f(1) = 3$, $f^{-1}(3) = 1$

6 Let $f(x) = x^3 + 3x + 1$. Find $\frac{d}{dx}\left(f^{-1}(x)\right)$

$y = f^{-1}(x)$, $f(y) = x$, $y^3 + 3y + 1 = x$, $(3y^2+3)\left(\frac{dy}{dx}\right) = 1$, $\frac{dy}{dx} = \frac{1}{3y^2+3}$

7 If $y = \ln|\sec x|$ then $\frac{dy}{dx} =$ $\underline{\frac{1}{\sec x} \sec x \tan x = \tan x}$

8 If $y = 2^{\sin x}$ then $\frac{dy}{dx} =$ $\underline{2^{\sin x} \cos x \ln 2}$

~~9 Find x if $\ln\left(\frac{5}{x}\right) + \ln(2x^3) = \ln(40)$~~