UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 249 — L07 WINTER 2005
MIDTERM EXAMINATION

Instructor: A.A. Bruen | 50 Minutes | Date: March 4, 2005

INSTRUCTIONS:

Marks

PART A

[18] 1. (a) State the Intermediate Value Theorem

Let $f(x)$ be continuous on $[a,b]$. Then, if $f(a), f(b)$ are of opposite sign there is at least one x_0 in between a and b with $f(x_0) = 0$

(b) Show that the equation $x^3 = x^2 - x + 1$ has at least one solution in the interval $[0, 2]$.

Let $f(x) = x^3 - (x^2 - x + 1) = x^3 - x^2 + x - 1$

Then, $f(0) = -1 < 0$, $f(2) = 2^3 - 2^2 + 2 - 1 = 5 > 0$

By 1(a) there exists x_0 in $[0,2]$ with $f(x_0) = 0$

Since $f(x) = x^3 - (x^2 - x + 1)$ there is at least one x_0 in $[0,2]$ such that

$$x_0^3 = x_0^2 - x_0 + 1$$

[20] 2. (a) Calculate the local linear approximation of $f(x) = \sqrt{9+x}$ for values of x close to 0.

$$f(x) \doteq f(a) + f'(a)(x-a) \qquad (1)$$

$$f(x) = \sqrt{9+x}, \quad a = 0, \quad f(0) = \sqrt{9} = 3$$

$$f'(x) = \frac{1}{2\sqrt{9+x}}, \quad f'(0) = \frac{1}{2\sqrt{9+0}} = \frac{1}{6}$$

From (1)

$$\sqrt{9+x} \doteq 3 + \frac{1}{6}x \qquad (2)$$

(b) Use your answer in (a) to estimate $\sqrt{10}$.

From (2) with $x = 1$ we get

$$\sqrt{10} \doteq 3 + \frac{1}{6} = \frac{19}{6}$$

[10] 3. (a) Find $f'(x)$ if $f(x) = \dfrac{x^2 - 2x + 3}{x^3 + x^2 - 7}$.

$$f'(x) = \frac{(2x-2)(x^3+x^2-7) - (x^2-2x+3)(3x^2+2x)}{(x^3+x^2-7)^2}$$

[10] (b) Find $f'(x)$ if $f(x) = \sin\left(\sqrt{x^2+x}\right)$.

$$f'(x) = \cos\left(\sqrt{x^2+x}\right) \frac{1}{2\sqrt{x^2+x}} (2x+1)$$

[12] 4. Let f be a differentiable function such that $f(1) = 4$, $f'(1) = 7$. Find the equation of the tangent line to the curve $y = f(x)$ at $x = 1$.

$(1, f(1)) = (1, 4)$

slope $= f'(x)\big|_{x=1} = f'(1) = 7$

Equation of tangent is

$y - 4 = 7(x - 1)$

$y = 7x - 3$

PART B – No Work Required. Only answers will be graded.

[8] 5. If $f(x) = x^3$, which of the following limits equals $f'(2)$?

(a) $\lim_{h\to 0} \dfrac{(2+h)^3 - 8}{h}$

(b) $\lim_{x\to 2} \left(\dfrac{x^3 - 8}{x}\right)$

(c) $\lim_{h\to 0} \dfrac{(2+h)^3 - h^3}{h}$

(d) None of the above.

[9] 6. Which of the following is $\dfrac{dy}{dx}$ given that $y + \cos y = x$.

(a) $\dfrac{dy}{dx} = \dfrac{1}{\sin y}$

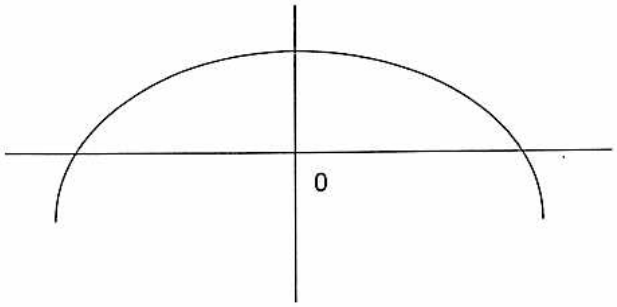
(b) $\dfrac{dy}{dx} = -\sin y$

(c) $\dfrac{dy}{dx} = \dfrac{1}{1 - \sin y}$

(d) $\dfrac{dy}{dx} = 1 - \sin y$

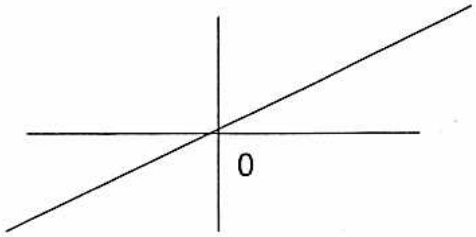
[5] 7. $\lim_{x\to 0} \dfrac{|x+5| - 5}{x} =$ 1

8. Let $f(x)$ be the parabola as in the diagram below.

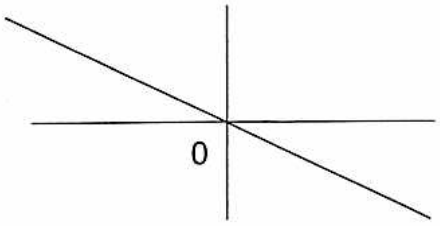


Which of the following graphs is the graph of $f'(x)$?

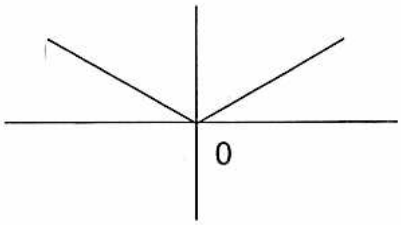
(a)



(b)



(c)



(d)