UNIVERSITY OF CALGARY

Faculty of Science
Department of Mathematics & Statistics

MAT 249 - Midterm

Winter 2007 - Lecture 08 - L. Nguyen Van Thé

Friday, March 16, 2007

Duration: 50 minutes - Points: 20

ID NUMBER:

INSTRUCTIONS:
I. Do not put your name on this page. Do put it together with your student number in the provided space at the top of every left page.
II. **Show all your work** and **write words**, not only formulas. Use the left pages for rough work and clearly insert the main steps and answers in the provided space.
III. Calculators and documents are **not allowed**.
IV. There are 15 questions and 15 pages to this exam. For every question, you are allowed to use the result of a previous question even if you did not find the answer to that previous question.
V. Time allowed is 50 minutes.
VI. Good luck.

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Part 1:	**/3**
Part 2:	**/4**
Part 3:	**/8**
Part 4:	**/5**
Total grade:	**/20**

Part 1: [3 pts]

Consider the functions u and v defined by:

$$u(x) = \cos\left(x^3 + 3x - 4\right) \qquad v(x) = \frac{x^3 + 3x - 4}{x^2 + 1}.$$

1- [1pt] Explain why u is differentiable on $\mathbb{R}$ and compute u'.

2- [1pt] Explain why u' is differentiable on $\mathbb{R}$ and compute u''.

3- [1pt] Explain why v is differentiable on $\mathbb{R}$ and compute v'.

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Part 2: [4 pts]

Consider the polynomial function P defined by:

$$P(x) = 2x^2 - x - \frac{2}{9}.$$

4- [1pt] Compute the discriminant Δ of P coming from the quadratic formula and express it as the square of a fraction.

5- [2pts] Using the result of question 4 and the quadratic formulas, show that the roots of P are $(-\frac{1}{6})$ and $\frac{2}{3}$ (You will not get the points of that question if you do not use the quadratic formulas).

6- [1pt] Study the sign of P. Use a sign table to express your result.

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Part 3: [8 pts]

Let f be the function defined by:

$$f(x) = \begin{cases} \frac{3x^2+x+\frac{1}{12}}{2x^2-x-\frac{2}{9}} & \text{if } x \neq -\frac{1}{6} \\ 0 & \text{if } x = -\frac{1}{6}. \end{cases}$$

7- [1pt] Compute the domain of f.

8- [2pt] Show that f has a finite limit at $+\infty$ and compute that limit. Use that result to show that the graph of f has a horizontal asymptote and give an equation for that asymptote.

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9- [1pt] Show that f has a finite limit at $(-\frac{1}{6})$ and compute that limit.

10- [1pt] Is f continuous at $(-\frac{1}{6})$? Explain your answer.

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11- [3pts] Compute $\lim_{x \to \frac{2}{3}^-} f(x)$ and $\lim_{x \to \frac{2}{3}^+} f(x)$. Use those results to show that the graph of f has a vertical asymptote and give an equation for that asymptote.

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Part 4: [5 pts]

Consider the polynomial function Q defined by:

$$Q(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - \frac{2}{9}x + \frac{4}{9}.$$

12- [1pt] Explain why Q is differentiable on $\mathbb{R}$ and compute its derivative. How does Q' relate to P from part 2?

13- [1pt] Using the result of question 6, find the intervals of increase and decrease of Q. Express your result in a table.

14- [2pts] Does Q have local extrema? If so, indicate at which points and whether they are minima or maxima.

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15- [1pt] Show that Q has a root in $\mathbb{R}$. You are **not** asked to compute that root, you only need to show that there is one. If you are using a theorem to prove that result, state precisely which one and mention explicitly how you apply it.