THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
FINAL EXAMINATION
MATHEMATICS 251 L01 - Winter, 2009

Time: Two hours

I.D. NUMBER	SURNAME	OTHER NAMES

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. **A student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.**

EXAMINATION RULES

1. Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
2. No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
3. All enquiries and requests must be addressed to supervisors only.
4. **Candidates are strictly cautioned against**:
   (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
   (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
   (c) making use of calculators and/or portable computing machines not authorized by the instructor;
   (d) leaving answer papers exposed to view;
   (e) attempting to read other students' examination papers.

   The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.
5. Candidates are requested to write on both sides of the page, unless the examiner has asked that the left half page be reserved for rough drafts or calculations.
6. Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
7. Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set.
8. The candidate is to write his/her name on each answer book as directed and is to number each book.
9. During the examination a candidate must report to a supervisor before leaving the examination room.
10. Candidates must stop writing when the signal is given. Answer books must be handed to the supervisor-in-charge promptly. Failure to comply with these regulations will be cause for rejection of an answer paper.
11. If during the course of an examination a student becomes ill or receives word of domestic affliction, the student must report at once to the supervisor, hand in the unfinished paper and request that it be cancelled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counsellor so that subsequent application for a deferred examination is supported by a completed Physical/Counsellor Statement form. Students can consult professionals at University Health Services or Counselling and Student Development Centre during normal working hours or consult their physician/counsellor in the community. **Once an examination has been handed in for marking a student cannot request that the examination be cancelled for whatever reason. Such a request will be denied. Retroactive withdrawals will also not be considered.**

Question	Total Marks	Actual Marks
1	12	
2-6	25	
7	16	
8	16	
9	12	
10	12	
11	16	
12	23	
13	16	
14	18	
15	14	
Total	180	

NOTE: No formula sheets or calculators are allowed.

Part A: No explanation needed.

1. In the space provided, enter T if the statement is always true, otherwise enter F: [12]

______(a) If a function $f(x)$ is continuous on a closed interval $[a, b]$ and k is a number between $f(a)$ and $f(b)$, then $k = f(c)$ for some number $c \in [a, b]$.

______(b) If a function $f(x)$ is continuous on a closed interval $[a, b]$ and c is a number between a and b, then $f(c)$ lies between $f(a)$ and $f(b)$.

______(c) The Mean-Value Theorem states that if a function $f(x)$ is continuous in $[a, b]$, then there is at least one point c in (a, b) such that $f(x)$ is differentiable at c and $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

______(d) The Extreme-Value Theorem says that if $f(x)$ is continuous on a finite closed interval $[a, b]$, then $f(x)$ has both an absolute minimun and an absolute maximun on $[a, b]$.

______(e) If a function $f(x)$ has an absolute minimum on an open interval at a point x_0, then $f'(x_0) = 0$ or $f'(x)$ is undefined at x_0.

______(f) If a function $f(x)$ is at least twice differentiable at x_0 with $f'(x_0) = 0$ and $f''(x_0) < 0$, then $f(x)$ has a relative minimum at x_0.

In Problems 2-6 ([5] each), choose a correct answer only (there is only one correct answer).

2. Let $f(x) = \begin{cases} (a\,x-3)^2, & \text{if } \; x<1 \\ 1, & \text{if } \; x=1 \\ x^3 - a\,x + 2 & \text{if } \; x>1 \end{cases}$. For what value of $a \in \mathbb{R}$ does $f(x)$ have a removable discontinuity at $x = 1$?

   (a) 0;

   (b) 1;

   (c) 2;

   (d) 3;

   (e) None of the above.

3. The derivative of $f(x) = (1+x^2)^{\ln(x)}$ is:

   (a) $2x(1+x^2)^{\ln(x)-1}$;

   (b) $\frac{1}{x}\ln(1+x^2) + \frac{2x}{1+x^2}\ln(x)$;

   (c) $(1+x^2)^{\ln(x)}\Big(\frac{1}{x}\ln(1+x^2) + \frac{2x}{1+x^2}\ln(x)\Big)$;

   (d) $(1+x^2)^{\ln(x)-1}$;

   (e) None of the above.

4. Let

$$F(x) = \int_1^{e^x} \frac{t^2}{\sqrt{\ln(t)+6}}\,dt.$$

   Then the derivative $F'(x) = \dfrac{d}{dx}F(x)$ of $F(x)$ is

   (a) $\dfrac{(e^x)^3}{\sqrt{x+6}}$;

   (b) $\dfrac{x^2}{\sqrt{\ln(x)+6}}$;

   (c) $\dfrac{e^{2t}e^x}{\sqrt{t+6}}$;

   (d) $\dfrac{t^2}{\sqrt{\ln(t)+6}}$;

   (e) None of the above.

5. A ladder 13 m long leans against the wall of a building. The bottom of the ladder slides away from the wall at a rate of 2 m/s. At which rate is the top of the ladder moving down the wall when the bottom is 5 m from the wall?

   (a) $\frac{6}{5}$ m/s;

   (b) $\frac{5}{12}$ m/s;

   (c) $\frac{5}{6}$ m/s;

   (d) 12 m/s;

   (e) This problem cannot be solved since the height of the building is not given.

6. A closed rectangular container with a square base is to have a volume of 2000 cm^3. It cost twice as much per cm^2 for surface area of the top and bottom as it does for the four sides. The dimensions of the container of least cost are:

   (a) 20 cm wide and 10 cm high;

   (b) 10 cm wide and 20 cm high;

   (c) 20 cm wide and 30 cm high;

   (d) 30 cm wide and 20 cm high;

   (e) The container of least cost is a cube, but it is impossible to find its dimensions.

Part B: Show your work.

7. Evaluate the following limits:

   (a) $\lim_{x \to 0} \dfrac{xe^x - \sin(x)}{e^x + e^{-x} - 2}$ [8]

   (b) $\lim_{x \to 2} \dfrac{x^2 - 3x + 2}{x^2 - 6x + 8}$ [8]

8. Find the derivatives of the following functions: (Do not simplify)

   (a) $f(x) = \dfrac{e^{2x}\cos(x)}{\sqrt{x} + \sin(x)}$ [8]

   (b) $f(x) = \tan^2(x + \ln(x))$ [8]

9. Find the equation of the tangent line to the curve $y^2e^x + x^2e^y = 1$ at the point $(1, 0)$. [12]

10. Find the absolute maximum and minimum values of the function $f(x) = x^3 - 12x$ on the interval $[-3, 3]$, and state where those values occur. [12]

11. (a) Find the local linear approximation of $f(x) = e^{-x}$ at $x_0 = 0$. [8]

(b) Use the approximation from part (a) to estimate the value of $e^{-0.12}$. [8]

12. Given $f(x) = xe^{-x^2/2}$, $f'(x) = (1 - x^2)e^{-x^2/2}$, $f''(x) = (x^3 - 3x)e^{-x^2/2}$, $\lim_{x\to-\infty} f(x) = 0 = \lim_{x\to+\infty} f(x)$.

    (a) Find all **closed** intervals where $f(x)$ is increasing or decreasing. [6]

    (b) Find all **open** intervals where $f(x)$ is concave up or concave down. [6]

    (c) Find the (x, y)-coordinates of all relative extrema (relative maxima and relative minima) and inflection points, if any. [10]

    (d) Find the asymptotes of the graph of $f(x)$. [1]

13. Assume that the position of a particle moving along an s-axis is given as a function of the time t by $s(t) = t^3 - 3t^2 - 6t + 8$ for $t > 0$.

(a) Find the instantaneous velocity, $v(t)$, and acceleration, $a(t)$. [3]

(b) Explain why is the particle stopped at a certain time between $t_1 = 1$ and $t_2 = 4$? (Hint: Use Rolle's Theorem) [3]

(c) Find all intervals for $t > 0$, when the particle is speeding up and when it is slowing down. [7]

(d) Find the average velocity, v_{av} between the time interval $t_1 = 2$ and $t_2 = 3$. [3]

14. Evaluate the following integrals:

(a)

$$\int \left(\sqrt[3]{x} - \cos(x) + \frac{3}{x} - 5e^x \right) dx;$$

[8]

(b)

$$\int x^2 \sqrt{x-1}\, dx.$$

[10]

15. (a) Find the **net signed** area of the region bounded by the graph of the function $f(x) = -3x^2 + 4x + 4$, the x-axis and the vertical lines $x = 0$ and $x = 3$. [6]

(b) Find the **total** area of the region bounded by the graph of the function $f(x) = -3x^2 + 4x + 4 = (2 - x)(3x + 2)$, the x-axis and the vertical lines $x = 0$ and $x = 3$. (Do not simplify) [8]

End of Examination