

UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
MATHEMATICS 249 — L02 F 2007

MIDTERM EXAM [October 31, 2007 (Wednesday)]

**Time: 60 minutes. PLEASE write your Name on the very last page.
NO CALCULATORS.
Total Marks = 100. Work all problems. Marks are shown in brackets.**

Student ID: _____

[Marks]

- [10] 1. Solve the following inequality for $x \in \mathbb{R}$ and express the solution set in interval notations:

$$\frac{x-1}{x-3} \leq \frac{x-3}{x-1}.$$

[12] 2. Find the derivatives $f'(x)$ of the following functions:

(a) $f(x) = \cos(4x) \sin^4(x)$.

(b) $f(x) = \frac{\sin x}{1 + x^2}$

3. Let $f(x) = x^2 \sin(x)$.

[15] (a) Find the local linear approximation of $f(x)$ at the point $x_0 = \frac{\pi}{2}$.

[6] (b) Using the linear approximation from part (a) to estimate the value of $f(-\frac{\pi}{2})$, what is the error in this approximation.

- [15] 4. Find the equation of the tangent line to the graph of the curve

$$x^4 + y^4 = 2x^3y$$

at the point $(1, 1)$.

5. Let $f(x)$ be the function defined piecewise by

$$f(x) := \begin{cases} x^3 & \text{if } x \leq 1 \\ 3x - 2 & \text{if } x > 1. \end{cases} \quad (1)$$

[6] (a) What is the value of the limit $\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h}$?

[6] (b) What is the value of the limit $\lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h}$?

[8] (c) Is the function f differentiable at $x_0 = 1$? Explain your answer.

[6] (d) Is the function f continuous at $x_0 = 1$? Explain your answer.

- [6] 6. Prove that the equation $x^3 - 15x + 1 = 0$ has at least **two solutions** in the interval $[0, 4]$.

7. Evaluate the following limits:

(a) $\lim_{x \rightarrow +\infty} \sqrt{x^2 + 3x} - \sqrt{x^2 + x}$

(b) $\lim_{x \rightarrow +\infty} \frac{6x^3 - 2x^2 - 3x + 4}{5 - 2x + 3x^2 - 3x^3}$

Name:	Student ID:	Marks:
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