Old Midterm 1:

1. $(-\infty, 2) \cup (5, \infty)$.
2. (a) $\frac{2}{3}$;
   (b) 2.
3. (a) 1, (b) $\frac{2}{3} - \frac{a}{3}$, (c) $\frac{3}{2-a}$, (d) -1, (e) 5, (f) 2, (h) Any real number.
4. (a)
$$y' = \frac{(x^2+1)(\tan x + x\sec^2 x) - 2x^2 \tan x}{(x^2+1)^2};$$
   (b)
$$y' = \frac{1}{2}\Big((x^2+1)\sin^3 x\Big)^{-\frac{1}{2}}\Big(2x\sin^3 x + 3(x^2+1)\sin^2 x\cos x\Big).$$
5. (a) $L(x) = 2 - \frac{1}{4}(x-1)$;
   (b) $\sqrt{4.12} = f(0.88) \approx L(0.88) = 2 - \frac{1}{4}(0.88-1) = 2.03$;
   (c) 0.001.
6. (a) $y' = \frac{3x^2-y}{2y+x}$, (b) $y - 2 = \frac{5}{3}(x-2)$.
7. $(1, 2)$.

Old Miterm 2:

1. $(-\infty, 1) \cup [2, 3)$.
2. (a) $f'(x) = -4\sin 4x \sin^4 x + 4\cos 4x \sin^3 x \cos x$;
   (b) $f'(x) = \frac{(1+x^2)\cos x - 2x\sin x}{(1+x^2)^2}$.
3. (a) $L(x) = \frac{\pi^2}{4} + \pi(x - \frac{\pi}{2})$;
   (b) $E(-\frac{\pi}{2}) = |f(-\frac{\pi}{2}) - L(-\frac{\pi}{2})| = \frac{\pi^2}{2}$.
4. $y = x$.
5. (a) 3, (b) 3, (c) the function $f(x)$ is differentiable at $x = 1$ because, $f'_-(1) = 3 = f'_+(1)$, and $f'(1) = 3$, (d) the differentiability of $f(x)$ implies its continuity.

6. $f(x) := x^3 - 15x + 1$ is everywhere continuous, so it has zeros (by the IVT) on any closed interval where it changes sign. Next, $f(0) = 1 > 0$, $f(1) = -13 < 0$, $f(3) = -17 < 0$ and $f(4) = 5 > 0$. Therefore, $f(x)$ has roots in $[0, 1]$ and $[3, 4]$, and the two roots are contained in the interval $[0, 4]$.

7. (a) 1, (b) -3.