THE UNIVERSITY OF CALGARY
MATHEMATICS 249
FINAL EXAMINATION, FALL 2004
TIME: 2 HOURS

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11	
Total (max. 65)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [65]. THIS EXAM HAS 8 PAGES INCLUDING THIS ONE.

[5] 1. Find $\dfrac{d}{dx}\left[\sin\left(\dfrac{x^3}{e^x - 5x}\right)\right]$.

[5] 2. Find $\dfrac{d}{dx}\Big(x^{4/3}(\sqrt{3+x} + 3 + x^2)\Big)$.

[6] 3. USE THE DEFINITION OF DERIVATIVE to find $\dfrac{d}{dx}\left(\dfrac{-2}{x}\right)$.

[6] 4. Use implicit differentiation to find $\dfrac{dy}{dx}$ where $\cos(x-y^2)=\cos x-\cos(y^2)$.

[8] 5. Let $f(x) = \sqrt{4 - \ln x}$.

(a) Find the domain of the function $f(x)$.

(b) Find and simplify the equation of the tangent line to the graph of $f(x)$ at the point where $x = 1$.

[9] 6. For the function $f(x) = \dfrac{x^3}{x+2}$, you are given that $f'(x) = \dfrac{2x^3 + 6x^2}{(x+2)^2}$.

(a) Find the domain of $f(x)$.

(b) Find the critical numbers; the intervals of increase and decrease; and all local maximum and minimum values of $f(x)$.

(c) Find $\lim\limits_{x \to -2^+} f(x)$ and $\lim\limits_{x \to -2^-} f(x)$. Give reasons.

[5] 7. For the function $f(x) = 6 + 3x^2 - 2x^3$, find the intervals on which $f(x)$ is concave up and where it is concave down. Then find all points of inflection.

[5] 8. Find constants a and b so that the function

$$f(x) = \begin{cases} a - x^2 & \text{if } x \leq 1, \\ bx^3 & \text{if } x > 1 \end{cases}$$

is both continuous and differentiable at $x = 1$.

[5] 9. Find and simplify $\displaystyle\int_1^2 \frac{3}{2x^2}\,dx$.

[5] 10. Find and simplify $\displaystyle\int x^2 \sec^2(x^3-2)\,dx$.

[6] 11. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the curve $y = 20 - x^4$, as shown in the diagram.

[65]