

THE UNIVERSITY OF CALGARY
MATHEMATICS 249 L04
FINAL EXAMINATION, FALL 2009
TIME: 2 HOURS

NAME _____ ID _____ Section _____

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12	
Total (max. 75)	

SHOW ALL WORK. SIMPLIFY ALL ANSWERS AS MUCH AS POSSIBLE. NO CALCULATORS PLEASE.

THE MARKS FOR EACH PROBLEM ARE GIVEN TO THE LEFT OF THE PROBLEM NUMBER. TOTAL MARKS [75]. THIS EXAM HAS 8 PAGES INCLUDING THIS ONE.

[5] 1. Find $\lim_{x \rightarrow -1} \left(\frac{1}{x+1} + \frac{2}{x^2-1} \right)$. Do not use l'Hôpital's Rule.

[5] 2. Find $\lim_{x \rightarrow 3} \left(\frac{\ln(x^2 - 8)}{\ln(x - 2)} \right)$.

[5] 3. Find $\frac{d}{dx} \left(\frac{x^4}{2x - \tan x} \right)$.

[5] 4. Find $\frac{d}{dx} (\sin(x^{1/2} \cos x))$.

[6] 5. USE THE LIMIT DEFINITION OF DERIVATIVE to find $\frac{d}{dx} \left(\frac{2}{4-x} \right)$.

[6] 6. Use implicit differentiation to find the slope of the tangent line to the curve given by $x + y = e^x - e^{3y} + 4$ at the point $(3, 1)$ lying on this curve.

[14] 7. For the function $f(x) = \frac{x^2}{2-x}$, you are given that

$$f'(x) = \frac{x(4-x)}{(2-x)^2} \quad \text{and} \quad f''(x) = \frac{8}{(2-x)^3}.$$

(a) Find the critical points of $f(x)$.

(b) Find the intervals of increase and decrease of $f(x)$. Use them to determine whether each critical point in part (a) is a relative maximum, relative minimum, or neither.

(c) Find the absolute maximum and absolute minimum of $f(x)$ for x in the interval $[-1, 1]$.

(d) Find the intervals where $f(x)$ is concave up and where it is concave down.

[6] 8. The function f is defined by
$$f(x) = \begin{cases} \frac{5}{x} & \text{if } x < -1, \\ \frac{a}{x^2 + 1} & \text{if } x \geq -1. \end{cases}$$

(a) Find the constant a so that the function f is continuous at $x = -1$.

(b) For the value of a found in part (a), is f differentiable at $x = -1$? Explain.

[6] 9. An object moves on the number line according to the position function $s(t) = t^3 - 18t$.

(a) Find the velocity $v(t)$ and the acceleration $a(t)$ of the object.

(b) Find all times when the object is stopped.

[5] 10. Find and simplify $\int_0^1 (5x^{2/3} - 2x + 3) dx$.

[5] 11. Find and simplify $\int \frac{x}{7x^2 - 8} dx$.

[7] 12. (a) Find two nonnegative numbers x and y so that their sum is 4 and so that $x^3 + y^2$ is as small as possible.

(b) Find two nonnegative numbers x and y so that their sum is 4 and so that $x^3 + y^2$ is as large as possible.