**MATH 249**
**Worksheet #1 -Inequalities,lines, parabolas, circles.**

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**Recall:** "split points" are x-values for which the top or bottom equals zero "$\frac{0}{0}$";
they split the number line into sets;
"testing" means choose an x from each set , substitute into the expresion and find out if is positive or negative;
remember w ehave an expresion on one side and 0 on the other.

**For 1 a)**

$|2x + 1| \le |x - 2|$

Since |...| is always positive or zero we can square both sides and the sign of the inequality stays the same: $(2x + 1)^2 \le (x - 2)^2$ since $|...|^2 = (...)^2$

Now

$4x^2 + 4x + 1 \le x^2 - 4x + 4$ everything on one side: $3x^2 + 8x - 3 \le 0$

$(3x - 1)(x + 3) \le 0$,thus split points (roots)are: $x = -3, \frac{1}{3}$,

testing: $- -^{pos} -_{x=-4} - -_{-3} -_{x=0} -^{neg} - -_{\frac{1}{3}} - -^{pos} -_{x=1} - -$

so the solutions set is the closed interval $\left[-3, \frac{1}{3}\right]$

**b)**

$$\frac{3}{x+1} > \frac{1}{3} \quad \text{for } x \neq -1$$

everything on one side and common denominator: $\dfrac{3 \cdot 3 - (x + 1)}{(x + 1)3} > 0$

simplify:

$\dfrac{9 - x - 1}{3(x + 1)} > 0$ then $\dfrac{8 - x}{(x + 1)(3)} > 0$ ;

now,find the split points i.e. where the top or bottom equal zero

so split points are : $x = 8, -1$

testing: $- -^{neg} -_{x=-2} - -_{-1} - -^{pos} -_{x=0} - -_{8} - -^{neg} -_{x=10} - -$

solution set is the open interval $]-1, 8[$ or $(-1, 8)$.

**For 2)**

Complete the squares

$x^2 + 4x + y^2 - 2y = 11 \qquad x^2 + 4x + 4 + y^2 - 2y + 1 = 11 + 4 + 1$

$(x + 2)^2 + (y - 1)^2 = 16$ so $(x + 2)^2 + (y - 1)^2 = 4^2$

thus

$r = 4$ and the point $C(-2, 1)$ is the centre of the circle.

**For 3a)** $|x + 1| + 2 > 0$

Since |...| is always positive or zero $|x + 1| + 2$ is always positive for any $x$, so solution set is $(-\infty, +\infty)$.

**b)** $\dfrac{3}{x+1} \ge \dfrac{2}{x+3}$ for $x \neq -1, -3$

everything on one side and common denominator: $\dfrac{3(x + 3) - 2(x + 1)}{(x + 1)(x + 3)} \ge 0$

simplify the top:
$\dfrac{3x+9-2x-2}{(x+1)(x+3)} \geq 0$ then $\dfrac{(x+7)}{(x+1)(x+3)} \geq 0.$
So split points are : $x=-7,-3,-1$
testing: $\quad -^{neg} -_{-7} - - -^{pos} - - -_{-3} - -^{neg} - -_{-1} - -^{pos} - - -$
solution set: $\quad [-7,-3) \cup (-1,+\infty)$ .
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**For 4)**
For $\quad l_1 : 3x+2y=1 \quad l_2 : 2y-3x=0 \quad l_3 : 3x-2y=0$ and $l_4 : 2x-3y=2$
find the slopes: $m_1 = -\frac{3}{2} \quad m_2 = \frac{3}{2} \quad m_3 = \frac{3}{2} \quad m_4 = \frac{2}{3}$
so $\quad l_2 \| \ l_3$ since they have the same slope and $\quad l_1 \perp l_4$ since $m_1 \cdot m_4 = -1$.

**For 5 a)** $\quad \dfrac{1}{x+1} \leq 1+x$ for $x \neq -1$
everything on one side and common denominator: $\dfrac{1-(x+1)^2}{(x+1)} \leq 0$,simplify:
$\dfrac{1-x^2-2x-1}{(x+1)} \leq 0 \quad$ then $\quad \dfrac{-x(x+2)}{(x+1)} \leq 0.$
So split points are : $x=0,-2,-1$
testing $\quad -^{pos} - -_{-2} - -^{neg} -_{-1} - -^{pos} - - -_{0} - -^{neg} - -$
check the split points, we can have zero on the top but NOT in the denominator, thus the solution set is $\quad [-2,-1) \cup [0,+\infty)$ .
**b)**
$|3x-2| > 0$
Since $|...|$ is always positive or zero we have to elliminate zero :$3x-2=0$ for $x=\frac{2}{3}$
The solutions : $\quad x \neq \frac{2}{3} \quad$ or $\quad \left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, +\infty\right)$
**For 6)**
$\perp$ to x-axis means a vertical line so $x=-1$ ($y$ is any).
line parallel to the x-axis means a horizontal line so $y=3$ ($x$ any)
**For 7 a)**
$3x+7 > x^2$
Everything on one side: $0 > x^2-3x-7$ now find the roots ,
first discriminant $D=(-3)^2-4\cdot 1\cdot(-7)=9+28=37$,
so using the quadratic formula roots are
$x_1 = \frac{3-\sqrt{37}}{2} \doteq -1.54$ and $x_2 = \frac{3+\sqrt{37}}{2} \doteq 4.54$
Now testing : $- -^{pos} - -_{x_1} - -^{neg} - - -_{x_2} - -^{pos} -$
OR parabola open up and the vertex is below the x-axis
the solution set is $x \in (-1.54, 4.54) \quad$ between the roots $x \in (x_1, x_2)$
**b)**
$\dfrac{x}{2} < \dfrac{2}{x+3} \quad$ for $x \neq -3$
everything on one side and common denominator: $\quad \dfrac{x(x+3)-2\cdot 2}{2(x+3)} < 0$
simplify:
$\dfrac{x^2+3x-4}{2(x+3)} < 0$ then $\dfrac{(x+4)(x-1)}{2(x+3)} < 0. \quad$ So split points are : $x=-4,-3,1$

testing:$-^{neg}--_{-4}--^{pos}--_{-3}-^{neg}--_{1}-^{pos}--$

solution set: $(-\infty,-4)\cup(-3,1)$.

**For 8)**

$$x^2-6x+y^2=7 \qquad x^2+y^2+2y=15$$

Complete the squares :

$x^2-6x+9+y^2=7+9 \qquad x^2+y^2+2y+1=1+15$

So the equations are:

$(x-3)^2+y^2=16 \qquad x^2+(y+1)^2=16$

thus radii are the same $r=4$, the centres are points $(3,0)$ and $(0,-1)$.

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**For 9)** for $h\neq 0,7$

find the common denominator first, then use $\dfrac{\frac{a}{b}}{\frac{c}{d}}=\dfrac{a}{b}\cdot\dfrac{d}{c}=\dfrac{ad}{cb}$

$$\frac{\frac{3h+4}{7-h}-\frac{4}{7}}{\frac{25h}{7}}=\frac{\frac{7(3h+4)-4(7-h)}{7(7-h)}}{\frac{25h}{7}}=\frac{21h+28-28+4h}{7\,(7-h)}\cdot\frac{7}{25h}=\frac{1}{7-h}.$$

**For 10)** for $x\neq-1$ as in 9) and using $\dfrac{1}{\frac{c}{d}}=\dfrac{d}{c}$

$$\frac{1}{1+\frac{1}{x+1}}=\frac{1}{\frac{x+1+1}{x+1}}=\frac{x+1}{x+2} \qquad \text{and for } x\neq-2.$$

**For 11)**

factor out the polynomials

$$\frac{x^3+5x^2+6x}{12+x-x^2}=\frac{x(x^2+5x+6)}{-(x^2-x-12)}=\frac{x(x+3)\,(x+2)}{-(x-4)\,(x+3)}=-\frac{x\,(x+2)}{(x-4)}$$

for $x\neq-3,4$

Note:

we have to keep both conditions since all steps must make sense ;never zero at the bottom.

**For 12)**

factor out the polynomials ,then common denominator:

$$\frac{x}{x^2+x-2}-\frac{2}{x^2-5x+4}=\frac{x}{(x+2)(x-1)}-\frac{2}{(x-4)(x-1)}=\frac{x\,(x-4)-2\,(x+2)}{(x+2)(x-1)\,(x-4)}=$$

$$=\frac{x^2-6x-4}{(x+2)(x-1)\,(x-4)} \qquad \text{for} \qquad x\neq-2,1,4.$$

**For 13)**

$\dfrac{x}{x-1}<\dfrac{1}{x+1}$ for $x\neq\pm1$

everything on one side and common denominator: $\dfrac{x\,(x+1)-(x-1)}{(x-1)\,(x+1)}<0$

simplify $\dfrac{x^2+1}{(x-1)\,(x+1)}<0$ the top has NO real roots,always positive

thus only two split points $x=\pm1$

Now testing : $--^{pos}--_{-1}--^{neg}---_{1}--^{pos}-$

the solution set is $(-1,1)$.

**For 14)**

$\dfrac{x}{x-1}>\dfrac{4}{x}$ for $x\neq1,0$

everything on one side and common denominator:$\dfrac{x^2-4\left(x-1\right)}{\left(x-1\right)\left(x\right)}>0$

simplify $\qquad \dfrac{x^2-4x+4}{x\left(x-1\right)}>0 \qquad \dfrac{\left(x-2\right)^2}{x\left(x-1\right)}>0$

the top has a double root, always positive or zero

the split points $\qquad x=0,1,2 \qquad$ but only $x=0,1$ are switch points

Now testing :

$$-\,-^{pos}-\,-_{0}-\,-^{neg}-\,-\,-_{1}-\,-^{pos}-_{2}-^{pos}-\,-\,-$$

check the split points!!

the solution set is $\qquad \left(-\infty,0\right)\cup\left(1,2\right)\cup\left(2,+\infty\right).$