

The University of Calgary
 Department of Mathematics and Statistics
 MATH 249-01
 Quiz # 1W

Fall 2008

Name: _____ I.D.#: _____

1. Solve for x: (a) $\frac{8}{x} \geq \frac{x+10}{x+1}$; (b) $|2x+3| > -1$. [4]

2. Find the equation of the line perpendicular to the line $3x - 4y = 4$, passing through the center of the circle $x^2 + y^2 - y = 1$ [3]

3. Simplify and find all x for which the expressions are defined $\frac{x-3}{1 + \frac{6x}{x^2+9}}$. [3]

Solution

For 1a) for $x \neq -1, 0$

$$\frac{8}{x} - \frac{10+x}{x+1} \geq 0 \rightarrow \frac{8(x+1) - x(10+x)}{x(x+1)} \geq 0 \rightarrow \frac{8x+8-10x-x^2}{x(x+1)} \geq 0$$

$$\frac{8-2x-x^2}{x(x+1)} \geq 0 \text{ multiply by } -1 : \frac{x^2+2x-8}{x(x+1)} \leq 0 \rightarrow \frac{(x+4)(x-2)}{x(x+1)} \leq 0$$

split points $x = -4, -1, 0, 2$

testing $\begin{matrix} \text{pos} & & \text{neg} & & \text{pos} & & \text{neg} & & \text{pos} \\ x=-5 & - & x=-2 & - & x=1 & - & x=4 & - \end{matrix}$
 check the split points $x = 0, -1$ Not incl., $x = -4, 2$ incl
 so $x \in [-4, -1) \cup (0, 2]$

For 1b)

always $|\dots| \geq 0$ so bigger than any neg#, thus $x \in (-\infty, +\infty)$, any x

For 2)

the given line $3x - 4y = 4 \rightarrow y = \frac{3}{4}x - 1$ has the slope $m = \frac{3}{4}$

so any perpendicular line has to have slope $m_{\perp} = -\frac{4}{3}$

to find the center complete the square

$$x^2 + y^2 - 2 \cdot \frac{1}{2}y + \frac{1}{4} = 1 + \frac{1}{4} \rightarrow x^2 + (y - \frac{1}{2})^2 = \frac{5}{4} \quad C(0, \frac{1}{2})$$

and together $y = -\frac{4}{3}x + \frac{1}{2}$ or $8x + 6y = 3$

For 3)

using $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$ with $b = 1$

$$\frac{x-3}{1 + \frac{6x}{x^2+9}} = \frac{x-3}{\frac{x^2+9+6x}{x^2+9}} = \frac{x-3}{\frac{(x+3)^2}{x^2+9}} = \frac{(x-3)(x^2+9)}{(x+3)^2} \quad \text{for } x \neq -3 \text{ only.}$$