

The University of Calgary
 Department of Mathematics and Statistics
 MATH 249-02 Quiz #1R Winter 2011

1. Solve for x :

$$|x + 5| \leq |x - 1| \quad [2]$$

2. Solve for x $\frac{3}{x} \leq \frac{2x}{x-2}$. [3]

3. Find an equation of the line perpendicular to the line $2x - 5y = 1$ and passing through the vertex of the parabola $2x^2 + 4x - 1 = y$. [3]

4. Simplify $\frac{x}{2 - \frac{1}{x+3}}$ and state for which x the given expression is defined. [2]

SOLUTION

For 1)

Since both sides are positive or 0 we can square:

$$(x + 5)^2 \leq (x - 1)^2 \rightarrow x^2 + 10x + 25 \leq x^2 - 2x + 1$$

$$12x \leq -24 \rightarrow x \leq -2 \quad x \in (-\infty, -2]$$

For 2)

for $x \neq 0, 2$ $\frac{3}{x} - \frac{2x}{x-2} \leq 0 \rightarrow \frac{3(x-2) - 2x^2}{x(x-2)} \leq 0$

$$\frac{-2x^2 + 3x - 6}{x(x-2)} \leq 0 \quad \text{the polynomial on the top has the discriminant}$$

$$D = 3^2 - 4(-2)(-6) = 9 - 48 = -39 < 0 \text{ so no real roots}$$

the top is always negative and only split points are $x = 0, 2$

testing $\begin{array}{ccccccc} & - & - & \text{neg} & - & - & 0 & - & - & \text{pos} & - & - & 2 & - & - & \text{neg} & - & - \end{array}$

so the solution set is $x \in (-\infty, 0) \cup (2, +\infty)$.

For 3)

the slope of the given line $2x - 5y = 1$ is $m = \frac{2}{5}$

so the slope of a perpendicular line is $m_{\perp} = -\frac{5}{2}$

to find the vertex complete the square $2x^2 + 4x - 1 = y$

$$2(x^2 + 2x) = y + 1 \rightarrow 2(x^2 + 2x + 1) = y + 1 + 2 \text{ so}$$

$2(x + 1)^2 - 3 = y$ so the vertex is $V(-1, -3)$ and the line is

$$y = -\frac{5}{2}(x + 1) - 3 \quad y = -\frac{5}{2}x - \frac{11}{2} \text{ or } 2y + 5x = -11$$

For 4)

for $x \neq -3$ $\frac{x}{2 - \frac{1}{x+3}} = \frac{x}{\frac{2(x+3)-1}{x+3}} = x \cdot \frac{x+3}{2x+5} = \frac{x(x+3)}{2x+5}$

for $x \neq -3, -\frac{5}{2}$ (both).