LABS 1, Math 251, L08, Sept. 8, 2003

1. Open boxes are cut from sheets of cardboard that are 6 inches square by cutting small squares from the corners of side x and folding up the sides. Find the volume V of the box in terms of x. What is the physical restriction on x?
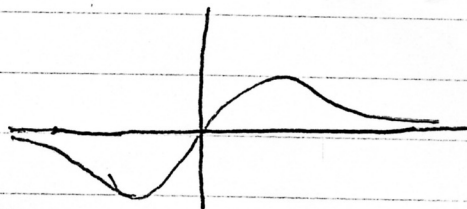
2. Let $g(x) = 3|x-2| - |x+1|$

   (a) Express the function in piecewise form

   (b) Sketch the graph of $g(x)$

3. Express the length L of a chord of a circle with radius 10 cm. as a function of θ

4. Let the graph of $y = f(x)$ be as in the following diagram.

   (a) Draw the graph of $y = |f(x)|$

   (b) Draw the graph of $y = f(|x|)$

   (c) Draw the graph of $y = |f(|x|)|$

5. Let $f(x) = x^2$, $g(x) = \sqrt{1-x}$

   (a) Find $(f \circ g)(x)$ and find its domain

   (b) Find $(g \circ f)(x)$ and find its domain

6. Sketch the graph of $y = |1-x^2|$

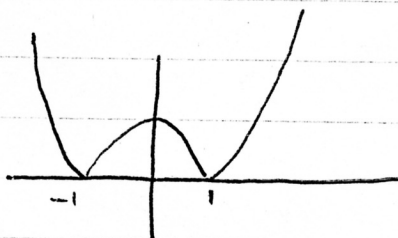7. Find the domain and range of $y = \dfrac{x+1}{x-1}$

Solutions

1. $V = x(6-2x)^2$, $x \geqslant 0$ ($x > 0$ is also correct)

2. $g(x) = \begin{cases} 7-2x, & x < -1 \\ 5-4x, & -1 \leq x < 2 \\ 2x-7, & x \geqslant 2 \end{cases}$ (The graph, which you should draw, is 3 segments of lines — joining up)
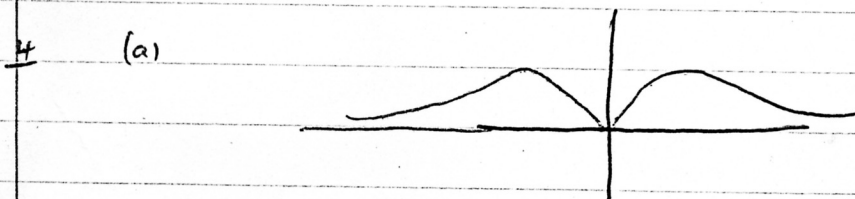
3. $L = 20 \sin\left(\frac{\theta}{2}\right)$

5. $(f \circ g)(x) = 1-x$, $x \leq 1$, $(g \circ f)(x) = \sqrt{1-x^2}$, $|x| \leq 1$

6.

7. $(f \circ g)(x) = \dfrac{1}{1-2x}$, $x \neq \frac{1}{2}, 1$

$(g \circ f)(x) = -\dfrac{1}{2x} - \dfrac{1}{2}$, $x \neq 0, 1$

4. (a)

(b) same

(c) same