MATH 251, LABS 2, SEPT 29, 2003

1 TRUE OR FALSE

(a) The 3 points determined by the intersection of the 3 lines $x+2y-3=0$, $2x-3y+4=0$ and $x-y+1=0$ form the vertices of a right angled triangle. ______

(b) If $f(x) = \frac{1}{x+1}$ then $f(f(f(f(f(0))))) = \frac{3}{5}$ ______

(c) The equation $\sin x = 3\cos x$ has at least one solution in $[0, 2\pi]$ ______

(d) The equation $x^3 = 15x-1$ has 3 solutions in $[-4,4]$ ______

(e) If f is continuous on $[0,1]$ with $0 \le f(x) \le 1$ then there must exist a number c in $[0,1]$ such that $f(c)=c$ ______

(f) If f, g are discontinuous at $x=c$ then $f+g$ must be discontinuous at $x=c$. ______

(g) If $f(x) = \begin{cases} \frac{x^3-8}{x^4-16}, & x \neq 2 \\ k, & x=2 \end{cases}$ and f is continuous at all values of x then $k = \frac{3}{8}$ ______

(h) If f_1 is even and f_2 is odd then $f_1 \circ f_2$ is odd ______

2 Find the domain of $g\circ f$ if $g(x) = \frac{4}{2x-8}$ and $f(x) = \sqrt{x^2-9}$.

3 Calculate the following limits

(a) $\lim\limits_{x \to -\infty} \left(\frac{\sqrt{3x^2+4}}{x} \right)$ (b) $\lim\limits_{x \to -3^+} \left(\frac{1-2x}{2x^2+5x-3} \right)$

4 Let $f(x) = \frac{|x+1| - x - 1}{2x^2+x-1}$

Find $\lim f(x)$ as (a) $x \to -1^-$ (b) $x \to 0$ (c) $x \to \infty$

5 $\lim\limits_{x \to 3} \left(\frac{\sqrt{x^2+2} - \sqrt{11}}{x-3} \right) =$ ______

Answers, Remarks

1 (a) FALSE. The slopes are $-\frac{1}{2}, \frac{2}{3}, 1$. For 2 lines to be perpendicular, the product of the slopes must be -1. (In this problem there is no need to calculate the vertices of the Δ).

(b) FALSE. $f(x) = \frac{1}{x+1}$, $f^2(x) = \frac{x+1}{x+2}$, $f^3(x) = \frac{x+2}{2x+3}$,

$f^4(x) = \frac{2x+3}{3x+5}$, $f^5(x) = \frac{3x+5}{3x+5+2x+3}$ (Pattern) $= \frac{3x+5}{5x+8}$

$\therefore f^5(0) = f(f(f(f(f(0))))) = \left.\frac{3x+5}{5x+8}\right|_{x=0} = \frac{5}{8} \neq \frac{3}{5}$

(c) Put $g(x) = \sin x - 3\cos x$. Now $g(0) = \sin 0 - 3\cos 0 = -3$. $g(2\pi) = \sin 2\pi - 3\cos 2\pi = -3$. So g does not have opposite signs at the end-points. However, $g\left(\frac{\pi}{2}\right) = 1$. Thus g has a root in $\left[0, \frac{\pi}{2}\right]$ and thus in $[0, 2\pi]$ ie $\sin x = 3\cos x$ for some x in $\left[0, \frac{\pi}{2}\right]$ and thus for some x in $[0, 2\pi]$ TRUE

(d) Put $g(x) = x^3 - (15x - 1) = x^3 - 15x + 1$. Note that $g(-4) < 0$, $g(0) > 0$, $g(1) < 0$, $g(4) > 0$ TRUE

(e) TRUE. If $f(0) = 0$ or $f(1) = 1$ we are done. Otherwise work with the continuous function $g(x) = f(x) - x$ and use the INT.

(f) FALSE e.g. let f be discontinuous at c and put $g(x) = -f(x) + 1$

(g) TRUE since $\lim_{x \to 2} \frac{x^3 - 8}{x^4 - 16} = \frac{3}{8}$

(h) FALSE, $f_1 \circ f_2$ is even since $(f_1 \circ f_2)(-x) = f_1(f_2(-x))$
$= f_1(-f_2(x))$ [since f_2 is odd] $= f_1 f_2(x)$ (since f_1 is even) $= f_1 \circ f_2(x)$

2 $(g \circ f)(x) = \frac{2(\sqrt{x^2 - 9} + 2)}{x^2 - 25}$, Domain$(g \circ f) = (-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

3 (a) $-\sqrt{3}$ (remember that $\sqrt{x^2} = |x|$) (b) $-\infty$

4 (a) $\frac{2}{3}$ (b) 0 (c) 0

e.g. in (b) x is close to 0, $x+1$ is close to 1 thus positive — so $|x+1| = x+1$ etc

5 $\frac{3}{\sqrt{11}}$ (Rationalize the ~~denominator~~ numerator)