Mathematics 251, L08, LABS 3, Oct. 14, 2003

RIGHT SIDE OF #3 should be $k \cos x$

1 Let $f(x) = \begin{cases} 2x+7, & x \le 1 \\ k x^2, & x > 1 \end{cases}$

Assume that $f(x)$ is continuous at $x = 1$

(a) Calulate $f'_-(1) = \lim_{x \to 1^-} \dfrac{f(x) - f(1)}{x-1}$ (b) $f'_+(1) = \lim_{x \to 1^+} \dfrac{f(x) - f(1)}{x-1}$

Is f differentiable at $x = 1$?

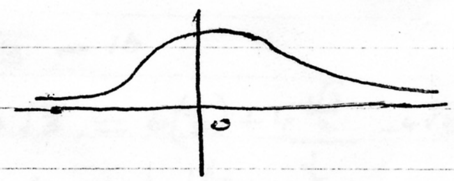
2 Find the equation of the tangent line to the curve $y = 6 \sin x + 4 \cot x - 4\sqrt{3}$ when $x = \frac{\pi}{6}$

3 Find k if $y = x \sin x$ is a solution of the equation $y'' + y = k \cancel{\sin} \cos x$

4 Find all values in the interval $[-2\pi, 2\pi]$ at which the graph of $f(x) = x + \cos x$ has a horizontal tangent.

5 Find $\frac{dy}{dx}$ at $x = \sqrt{\pi}$ if $y = \tan(4x^2)$

6 If the graph of $f(x)$ is as in the diagram sketch the graph of $f'(x)$



7 Find all values of x for which the tangent line to $y = 2x^3 - x^2$ is perpendicular to the line $x + 4y = 10$

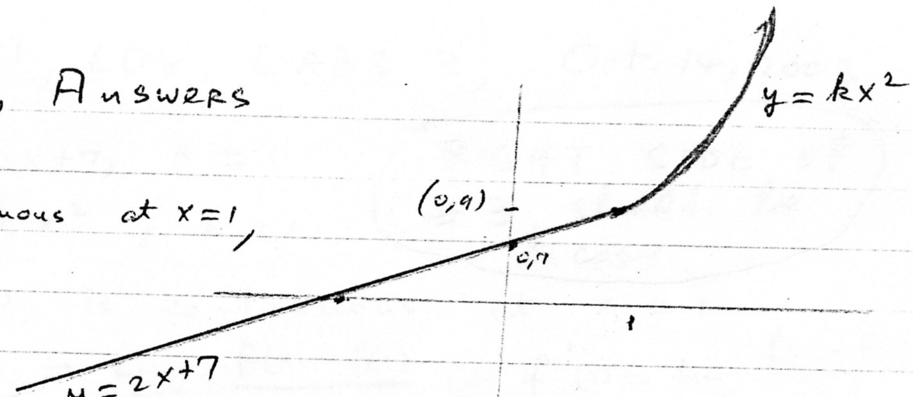
8 Find equations for two lines through the origin that are tangent to the curve $x^2 - 4x + y^2 + 3 = 0$

9 Using the definition of the derivative, find

$$\lim_{y \to 0} \left(\frac{\tan(x+y) - \tan x}{y} \right)$$

Solutions, Answers

1 Since $f(x)$ is continuous at $x=1$, we must have



$$\lim_{x\to 1^-}(f(x)) = \lim_{x\to 1^+}(f(x)) = \lim_{x\to 1}(f(x)) = f(1) = 9$$

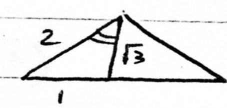
So $9 = k$ giving $k = 9$

(a) left derivative $= \frac{d}{dx}(2x+7) = 2$

(b) right derivative $= \frac{d}{dx}(kx^2)\Big|_{x=1,k=9} = 2kx\Big|_{x=1,k=9} = 18$

Since $2 \neq 18$, f is not diff. at $x=1$ (see p. 187 in text for left, right diff.)

2 $\frac{dy}{dx} = 6\cos x - 4\operatorname{cosec}^2 x$. When $x = \frac{\pi}{6}$

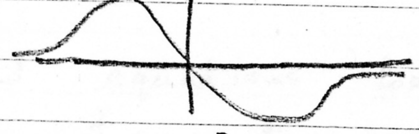


we get $\frac{dy}{dx}\Big|_{\frac{\pi}{6}} = 6\frac{\sqrt{3}}{2} - 4\frac{1}{\sin^2\frac{\pi}{6}} = 3\sqrt{3} - 16$

When $x = \frac{\pi}{6}$, $y = 6\sin\frac{\pi}{6} + 4\frac{\cos\frac{\pi}{6}}{\sin\pi/6} - 4\sqrt{3} = 6\left(\frac{1}{2}\right) + 4\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} - 4\sqrt{3} = 3$. So the point $\left(\frac{\pi}{6}, 3\right)$ is on the curve and the equation of the tangent line is $y - 3 = (3\sqrt{3} - 16)\left(x - \frac{\pi}{6}\right)$

3 $k = 2$. 4 $x = -\frac{3\pi}{2}, \frac{\pi}{2}$ 5 $8\sqrt{\pi}$

6



As in class, note that $f(x)$ is even so $f'(x)$ is odd

7 $x = 1, -\frac{2}{3}$ 9 This is $\frac{d}{dx}(\tan x) = \sec^2 x$

8 By implicit diffn. we get $\frac{dy}{dx} = \frac{2-x}{y}$. Let $P(x_0, y_0)$ be a point on the curve where a line through $(0,0)$ is tangent to the curve. Then $\frac{y_0}{x_0} = \frac{2-x_0}{y_0}$, so $y_0^2 = 2x_0 - x_0^2$. But $x_0^2 - 4x_0 + y_0^2 + 3 = 0$ since (x_0, y_0) is on the curve. Eliminating y_0^2 gives $x_0 = 3/2$ and $y_0 = \pm\frac{\sqrt{3}}{2}$. The slopes are $\frac{y_0}{x_0} = \pm\frac{\sqrt{3}}{3}$ and the equations are $y = \frac{\sqrt{3}}{3}x$ and $y = -\left(\frac{\sqrt{3}}{3}\right)x$