Mathematics 251, L08, LABS 4, Week of Nov. 3, 2003.

1 Show that $\lim_{x \to 0} (1+x)^{1/x} = e$

2 Evaluate $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$

3 Find $\lim_{x \to \infty} (e^x - x^2)$

4 Find $\lim_{x \to 1} \frac{\ln x}{x^4 - 1}$

5 Sketch the graph of a function f with the stated properties.

f is increasing on $(-\infty, \infty)$, has an inflection point at the origin and is concave down on $(0, \infty)$

6 Given that a is a constant and n a positive integer what can you say about the existence of inflection points of the function $f(x) = (x-a)^n$.

7 Show that $e^x \geq 1 + x$ if $x \geq 0$

8 If $x_1 < x_2$, must $e^{x_1} < e^{x_2}$?

9 True or False :

(a) If f, g are increasing on an interval I then so is f+g

(b) ______ I then so is f(x)g(x)

(c) ______ I ______ f∘g

Solutions, Answers

1 $y = (1+x)^{\frac{1}{x}}$, so $\ln y = \ln\left((1+x)^{1/x}\right) = \frac{1}{x}\ln(1+x)$

$\lim\limits_{x\to 0}(\ln y) = \lim\limits_{x\to 0}\left(\frac{\ln(1+x)}{x}\right) = \lim\limits_{x\to 0}\left(\frac{\frac{1}{1+x}}{1}\right)$ by L'HÔPITAL

$= 1$

Thus, $\ln y \to 1$ as $x \to 0$ so $e^{\ln y} \to e^{1}$ ie $y \to e$ as $x\to 0$

2 $\lim\limits_{x\to 0^+}\left(\frac{1}{x} - \frac{1}{\sin x}\right) = \lim\limits_{x\to 0^+}\left(\frac{\sin x - x}{x\sin x}\right) = 0$ (Apply L'HOPITAL TWICE)

3 $\lim\limits_{x\to\infty}\left(e^x - x^2\right) = \lim\limits_{x\to\infty} x^2\left(\frac{e^x}{x^2} - 1\right)$. But $\lim\limits_{x\to\infty}\left(\frac{e^x}{x^2}\right) = \infty$

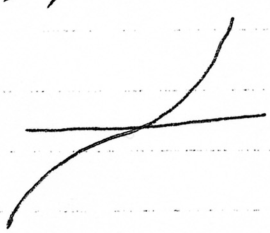
Thus $\lim\limits_{x\to\infty}\left(\frac{e^x}{x^2} - 1\right) = \infty$ so $\lim\limits_{x\to\infty} x^2\left(\frac{e^x}{x^2} - 1\right)$

$= \lim\limits_{x\to\infty}\left(e^x - x^2\right) = \infty$

4 $\lim\limits_{x\to 1}\left(\frac{\ln x}{x^4 - 1}\right) = \lim\limits_{x\to 1}\left(\frac{\frac{1}{x}}{4x^3}\right)$ by L'HÔPITAL

$= \frac{1}{4}$

5 e.g. $f(x) = x^3$



6 If n is odd then $(a,0)$ is a point of inflection. If n is even there is no inflection point

7 Let $h(x) = e^x - 1 - x$ for $x \geq 0$. Then $h(0) = 0$ and $h'(x) = e^x - 1 \geq 0$ for $x \geq 0$ so $h(x)$ is increasing

8 Yes: e^x is an increasing function

9 (a) True (b) True if f, g are ≥ 0 on I; otherwise not necessarily true (c) True