Mathematics 251, Quiz 2, Week of Oct. 6, 2003
(NO CALCULATORS OR NOTES PERMITTED)

NAME, ID: ______________________

SIGNATURE: ______________________

1 TRUE OR FALSE

(a) If $\lim_{x \to 2} (f(x)) = \infty$ then $f(x)$ cannot be continuous at $x = 2$

For continuity, $\lim_{x \to a} f(x) = f(a)$. But ∞ is not a number so we can't put $f(2) = \infty$ TRUE

(b) If a function is differentiable at $x = x_0$ then that function must be continuous at $x = x_0$ TRUE

(c) If g is a function such that $g(1) > 0$ and $g(2) < 0$ then there is an x_0 in $[1, 2]$ with $g(x_0) = 0$ FALSE

(d) If a function is continuous at $x = x_0$ then that

4 function must be differentiable at $x = x_0$ FALSE

For (c), g must be continuous to use the IMV

For (d), take $f(x) = |x|$ at $x = 0$. Then f is cont. but not diff. at $x = 0$

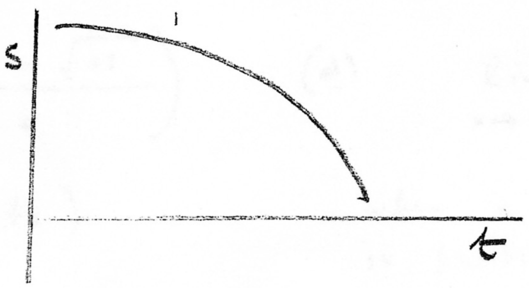
2 If $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1}, & x \neq 1 \\ k, & x = 1 \end{cases}$ and f is continuous

2 at $x = 1$ then $k =$ 2

$\lim_{x \to 1} \dfrac{x^2 - 1}{x - 1} = \lim_{x \to 1} (x + 1) = 2$

3 The accompanying diagram shows the position versus time curve of a particle moving on a straight

1 line, Determine whether the instantaneous velocity is increasing or decreasing with time,



decreasing

4 Let $f(x) = \frac{1}{x^2}$, $x \neq 0$. Find the average rate of change of $f(x)$ over the interval $[1,3]$

(2)

$$\frac{f(b)-f(a)}{b-a} = \frac{\frac{1}{3^2} - \frac{1}{1^2}}{3-1} = \frac{\frac{1}{9}-1}{2} = \frac{-\frac{8}{9}}{2} = -\frac{4}{9}$$

$-\frac{4}{9}$

5 Show that the equation $x^3 - 4x = -1$ has at least one solution in the interval $[1,2]$

(2) Put $g(x) = x^3 - 4x - (-1) = x^3 - 4x + 1$

now $g(1) = -2 < 0$ and $g(2) = 1 > 0$

Since g is continuous there is some x_0 in $[1,2]$ with $g(x_0) = 0$. Thus there is at least one x_0 in $[1,2]$ with $x_0^3 - 4x_0^2 = -1$

6 Calculate the following limits

(4)

(a) $\lim_{x \to 0} \frac{1-\cos(x^2)}{x^2}$

just like $\lim_{x \to 0} \left(\frac{1-\cos x}{x}\right)$

0

(b) $\lim_{x \to 0} \left(\frac{x}{x + \sin x}\right)$

$= \lim_{x \to 0} \left(\frac{1}{1 + \frac{\sin x}{x}}\right)$, dividing above and below by x.

$\frac{1}{2}$

(c) $\lim_{x \to 3} \left(\frac{\sqrt{x^2+2} - \sqrt{11}}{x^2 - 9}\right)$

(see lab sheets)

(15)

$\frac{1}{2\sqrt{11}}$

(d) $\lim_{x \to 0} \left(\frac{|2x+3| - 3}{x}\right)$

when x is close to 0, $2x+3$ is positive, $|2x+3| = 2x+3$,

$\frac{2x+3-3}{x} = \frac{2x}{x} = 2$

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