Mathematics 251, Quiz 3, Section L08, Week of Oct. 20, 2003

NAME, ID: ____________________

SIGNATURE ____________________

1 (a) If $\log_2(2x-1) = 4$ then $x = \underline{\frac{17}{2}}$

$2x-1 = 2^4 = 16, \; 2x = 17$

5

(b) If $\log_3\left(\frac{1}{x}\right) = -2$ then $x = \underline{9}$

$3^{-2} = \frac{1}{x}, \; \frac{1}{9} = \frac{1}{x}, \; x = 9$

(c) $\frac{d}{dx}\left(\tan(x^2)\right) = \boxed{[\sec^2(x^2)](2x)}$

(d) $\lim_{y \to 0} \left(\frac{\sec(x+y) - \sec(x)}{y} \right) = \underline{\frac{d}{dx}(\sec x) = \sec x \tan x}$

(e) Rewrite the following equations as an equation in $t\,(=e^x)$ if $e^{2x} - e^{-x} = e^x$

$e^{2x} - \frac{1}{e^x} = e^x, \; t^2 - \frac{1}{t} = t, \; t^3 - 1 = t^2$ $\qquad \underline{t^3 - t^2 - 1 = 0}$

2 Find $f'\left(\frac{4}{\pi}\right)$ if $f(x) = x^2 \sec\left(\frac{1}{x}\right)$

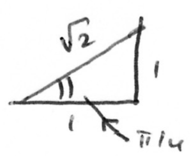
2

$$f'(x) = \frac{d}{dx}\left[x^2 \sec\left(\frac{1}{x}\right)\right] = 2x \sec\left(\frac{1}{x}\right) + x^2 \sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right)$$

$$= 2x \sec\left(\frac{1}{x}\right) - \sec\left(\frac{1}{x}\right)\tan\frac{1}{x}$$

$$f'\left(\frac{4}{\pi}\right) = 2x \sec\left(\frac{1}{x}\right) - \sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)\Big|_{\frac{4}{\pi}}$$

$$= \frac{8}{\pi}\sec\frac{\pi}{4} - \sec\frac{\pi}{4}\tan\frac{\pi}{4}$$



$$= \frac{8}{\pi}\frac{1}{\cos\left(\frac{\pi}{4}\right)} - \frac{1}{\cos\left(\frac{\pi}{4}\right)}\frac{\sin\frac{\pi}{4}}{\cos(\pi/4)}$$

$$= \frac{8}{\pi}\frac{1}{\frac{1}{\sqrt{2}}} - \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\pi} - \sqrt{2} = (\sqrt{2})\left(\frac{8}{\pi} - 1\right)$$

3 Find all values of x for which the tangent line to $y = 2x^3 - x^2$ is perpendicular to the line $x + 4y = 10$.

(2)

$\frac{dy}{dx} = 6x^2 - 2x$. This must be $\perp$ line with slope $-\frac{1}{4}$ (since $x+4y=10$ gives $4y = -x+10$, $y = -\frac{1}{4}x + \frac{10}{4}$)

Since the tangent line, having slope $6x^2-2x$, is $\perp$ line with slope $-\frac{1}{4}$

we get $(6x^2-2x)\left(-\frac{1}{4}\right) = -1$, $6x^2 - 2x = 4$, $6x^2-2x-4=0$

$3x^2 - x - 2 = 0$, $(3x+2)(x-1) = 0$, $x = -\frac{2}{3}$ or $x = 1$

4 Find the slope of the tangent line at $(-2, 1)$ to the curve $x^2 + xy + 2y^3 = 4$.

(3) We have $2x + \frac{d}{dx}(xy) + \frac{d}{dx}(2y^3) = 0$ so

$2x + y + x\frac{dy}{dx} + 6y^2\frac{dy}{dx} = 0$ or $2x + y + xy' + 6y^2y' = 0$

$\therefore \frac{dy}{dx}(x + 6y^2) = -2x - y$, $\frac{dy}{dx} = \frac{-2x-y}{x+6y^2}$

$$\left.\frac{dy}{dx}\right|_{(-2,1)} = \left.\frac{-2x-y}{x+6y^2}\right|_{(-2,1)} = \frac{4-1}{-2+6} = \frac{3}{4}$$

So the slope of the tangent line at $(-2,1)$ is $\frac{3}{4}$

5 Use an appropriate local linear approximation to estimate $26^{1/3}$, the cube root of 26.

(3)

$$f(x) \doteq f(x_0) + (x - x_0)\left(f'(x)\big|_{x=x_0}\right)$$

Take $x = 26$, $x_0 = 27$, $f(x) = x^{1/3}$, $f'(x) = \frac{1}{3}x^{-2/3}$

$x - x_0 = -1$ $\qquad f'(x_0) = \frac{1}{3}x_0^{-2/3}$

$$\therefore \sqrt[3]{26} \doteq \sqrt{27} - \frac{1}{3}\frac{1}{(\sqrt[3]{27})^2}$$

$$\therefore \sqrt[3]{26} \doteq 3 - \frac{1}{3}\frac{1}{3^2} = 3 - \frac{1}{3^3} = 3 - \frac{1}{27}$$

$$= \frac{80}{27}$$

(15)