Mathematics 251, L08, Quiz 4, Week of Nov. 17, 2003.

NAME, ID: ______________________

SIGNATURE ______________________

1 Let $f(x) = |2x^2-2|$. On which interval is the graph of f concave down?

(2) $f(x) = |2(x^2-1)| = |2|\,|x^2-1| = 2|x^2-1|$

The graph of f looks like

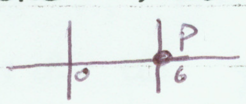
$-1 < x < 1$

2 Give a specific example of a function $f(x)$ and a value x_0 such that $f'(x_0) = 0$ but such that $f(x)$ does **not** have a local max. or min. at $x = x_0$

(1) $f(x) = x^3$ and $x_0 = 0$

3 If $f(6-x) = -f(6+x)$ then the point P is a point of symmetry for the graph of $f(x)$ where the coordinates of P are given by

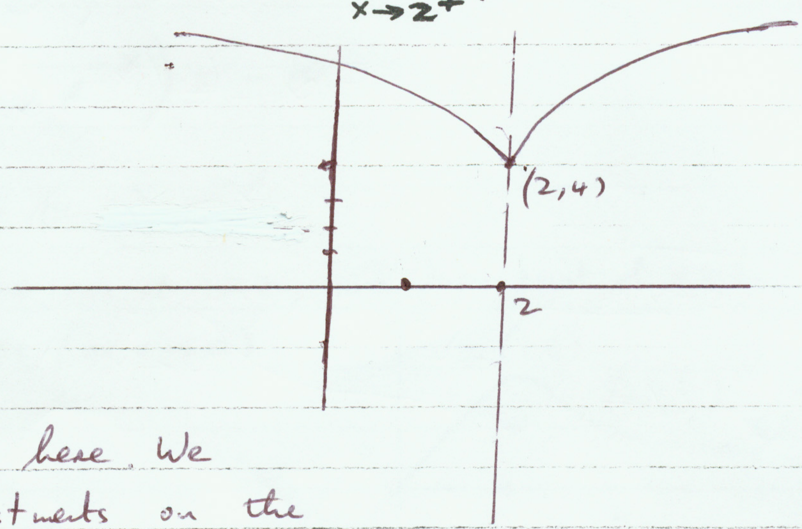
(1)



$P = (6,0)$

4 Draw a rough sketch of the graph of any function $f(x)$ satisfying all of the following properties:

(2) $f(2) = 4$, $f''(x) \overset{<0}{>} 0$ for all $x \neq 2$, $\lim_{x \to 2^+} (f'(x)) = \infty$ and $\lim_{x \to 2^-} (f'(x)) = -\infty$



Note: there is a "typo" here. We want $f''(x) < 0$. (Adjustments on the grading were made)

6

5 (a) $\lim\limits_{x\to 0} \dfrac{e^x - 1}{x} = $ 1 (b) $\lim\limits_{x\to 0} (1+2x)^{-\frac{3}{x}} = e^{-6}$

(2) (a) $\frac{0}{0}$, use L'HÔPITAL (b) $y = (1+2x)^{-\frac{3}{x}}$, $\ln y = -\frac{3}{x}\ln(1+2x) = -3\,\frac{\ln(1+2x)}{x}$

$\therefore \ln y \to \frac{-3}{1+2x}(2)\Big|_{x=0} = -6$, $y \to e^{-6}$

6 TRUE OR FALSE

(a) If f is even and $f'' \neq 0$ then $\dfrac{f}{f''}$ is even T

(4) f even $\Rightarrow f'$ odd $\Rightarrow (f')' = f''$ is even

(b) $\tan x > x$ for all x satisfying $0 < x < \frac{\pi}{2}$ T

$f(x) = \tan x - x$, $f(0) = 0$, $f'(x) = \sec^2 x - 1 = \frac{1}{\cos^2 x} - 1 > 0$ on $(0, \pi/2)$

(c) If $f(x), g(x)$ are increasing on I then $f(x)\,g(x)$ is also increasing on I e.g. $f(x) = x, g(x) = x$ on $(-1, 0)$ F

(d) If the differentiable functions f, g have a local (= relative) maximum at $x = a$ then $f - g$ also has a local maximum at $x = a$. F

No, e.g. Take $g = 2f$; then $f - g = -f$ which has a local min at $x = a$

7 Sketch the graph of $y = xe^{-x}$. You should indicate any asymptotes, relative extrema and points of inflection.

(3)

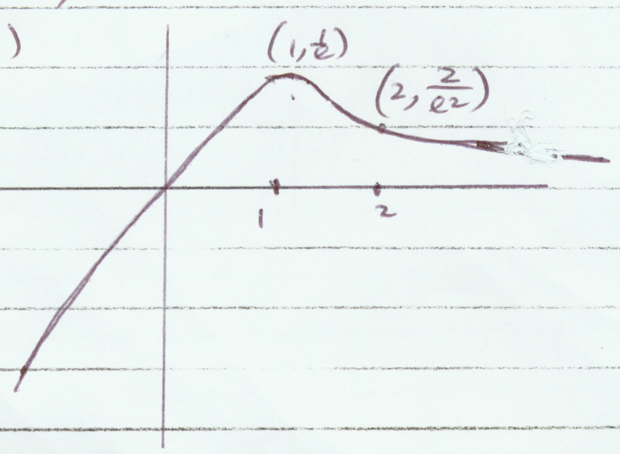
$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left(\frac{x}{e^x}\right) = 0$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(x e^{-x}\right) = -\infty$$

$f'(x) = (1-x)e^{-x}$, $f''(x) = (x-2)e^{-x}$

Critical point at $x = 1$, local max at $x = 1$, inflection point at $x = 2$

Horizontal asymptote $y = 0$ (as $x \to \infty$)



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