Mathematics 251, L08, Quiz 5, Week of Nov. 24, 200

NAME, I.D.

SIGNATURE

Let $f(x) = \dfrac{2x^2-8}{x^2-16}$ so that $f'(x) = \dfrac{-48x}{(x^2-16)^2}$ and

$f''(x) = \dfrac{48(16+3x^2)}{(x^2-16)^3}$

(a) Does the graph of f have any symmetries?

Yes, replacing x by $-x$ does not change $f(x)$ ie the graph of f is symmetric about the vertical axis

Symmetrical about vertical axis

(b) What are the vertical asymptote(s) of f? Lines $x=-4$, $x=4$

(c) What are the horizontal asymptote(s) of f? $y=2$

$\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 2$

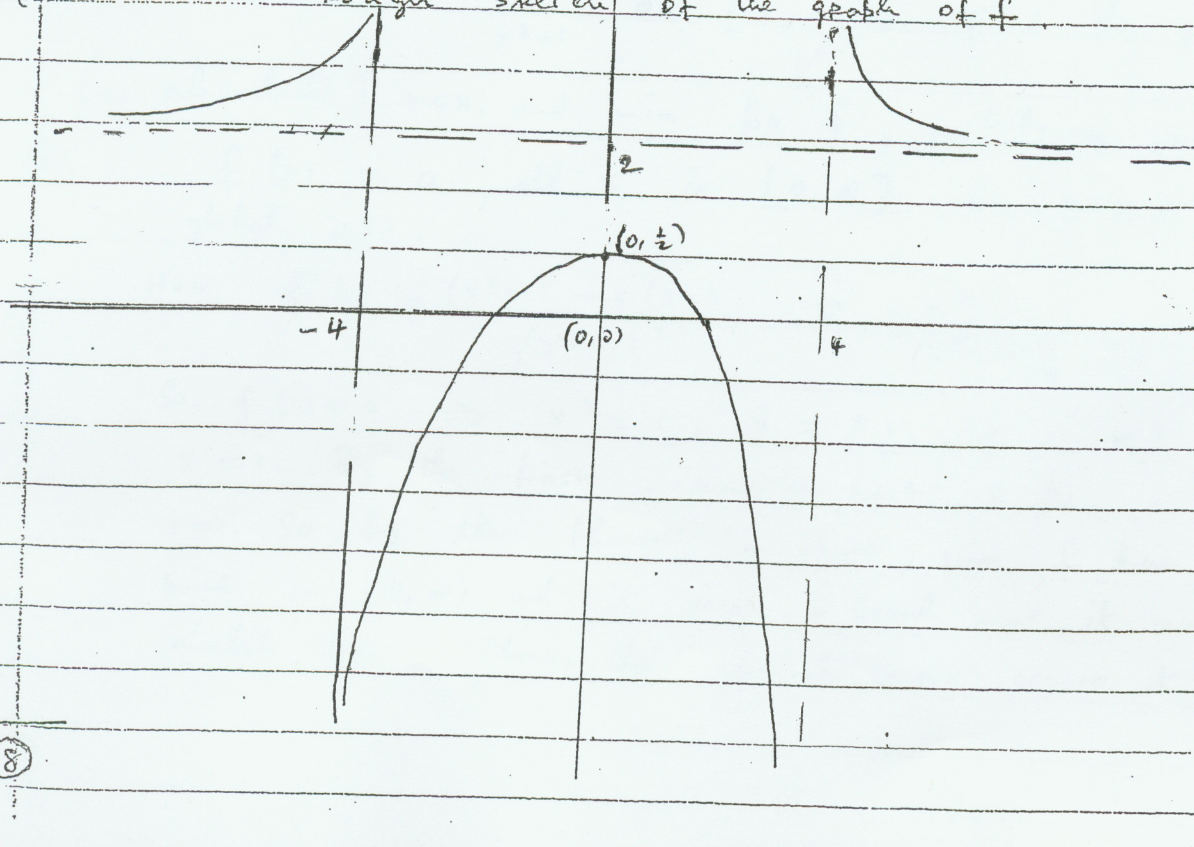
(d) For which values of x is f(x) decreasing?

$(0,4) \cup (4,\infty)$

(e) Where is the graph of f concave up? $(-\infty,-4) \cup (4,\infty)$

(f) The (x,y) coordinates for all local maxima of f are given by

(g) Draw a rough sketch of the graph of f.



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2. Explain precisely the first derivative test for local extrema.

(2) Suppose f is continuous at a critical number x_0.

(a) If $f'(x) > 0$ on an open interval extending left from x_0 and $f'(x) < 0$ on an open interval extending right from x_0 then f has a relative maximum at x_0

(b) If $f'(x) < 0$ on an open interval extending left from x_0 and $f'(x) > 0$ on an open interval extending right from x_0, f has a relative min. at x_0

(c) If $f'(x)$ has the same sign on an open interval extending left from x_0 and on an open interval extending right from x_0 then f does not have a relative extremum at x_0.

3. A particle is moving on a line in such a way that $s(t) = t^2 - 4t + 1$, $t \geq 0$. For which values of t

(1) is the particle moving in the negative direction?

$\frac{ds}{dt} = 2t - 4$, so $\frac{ds}{dt} < 0$ if

$2t - 4 < 0$, $2t < 4$, $t < 2$

$t < 2$

4 TRUE OR FALSE

(a) If $f(x)$ is defined on a finite interval I and f

(2) is continuous then f must have a global maximum on I (F e.g. $f(x) = x$ on $(0,1)$) (finite closed, yes) F

(b) If $f''(x_0) < 0$ then f has a local maximum at x_0. F

e.g. $f(x) = x^3$ has no local max or min anywhere but $f''(x) < 0$ for any $x < 0$ (For 2nd derivative test we need $f' = 0$)

5 Let $f(x) = \frac{x}{x^2+1}$ on $[0, \infty)$. Find the global (= absolute) max. and min for f, justifying your answers.

(2) $f(x) \geq 0$, all x in $[0, \infty]$ so $x = 0$ gives the global min.

Now $f'(x) = \frac{(x^2+1) - x(2x)}{(x^2+1)^2} = \frac{-x^2+1}{(x^2+1)^2}$

So, $f'(x) = 0 \Rightarrow x^2 = 1$, $x = \pm 1$. In $[0, \infty)$ we must have $x = 1$. By the first derivative test, f has a local max at $x=1$. So, by the theorem in class, since f has only 1 critical point in $[0, \infty)$ and it gives a local max, it must give a global max. Thus, the global max occurs at $x = 1$, $f(1) = \frac{1}{2}$

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