Final Exam (Fall 2002) Math 251, L08, 2003

1 (3 points) Find $\frac{dy}{dx}$ if $y = \frac{x}{x^2-4x+3}$

2 Find $\frac{dy}{dx}$ if $y = \frac{\tan^2 x}{\ln x - 2}$ (3 points)

3 If $y = \ln|x|$, $\frac{dy}{dx} =$ ——, If $y = 2^{\sin x}$, $\frac{dy}{dx} =$ —— [2 points]

4 If $f(x) = |9-x^3|$, $f'(2) =$ [1 point]

Do 5 OR 6

5 [5] Find the equation of the tangent line to the curve $x^3y + y^3x = 30$ at the point $(1,3)$ OR (ie 5 OR 6)

6 [7] Find the equations of the two lines tangent to the circle $x^2+y^2+4x+3=0$ that pass through the origin.

7 At what rate is the area of an equilateral triangle increasing if the side is 9 inches long and is increasing at a rate of .5 inches/second? The triangle stays equilateral during the expansion [5]

8 The sum of two non-negative numbers is 36. Find the numbers if the sum of their square roots is as large as possible, and show your work [5] OR (ie 8 OR 9)

9 A right circular cone of height 12 and base radius 6 contains another cone upside down within it. The two bases are parallel and the vertex of the smaller cone lies at the centre of the larger cone's base. What are the dimensions of the smaller cone that will give it the largest possible volume? [7]

10 Find $\lim_{x\to 0} \frac{2e^{\sin x} - 2x - 2}{3x^2 + x^4}$ [3].

11 (a) State the mean value theorem (b) Using the mean value theorem, show that if $f'(x) > 0$ on an interval I then f is increasing on I [3]

12 [6] Compute the following integrals (a) $\int_0^2 (2-x)^5\,dx$ (b) $\int \frac{x^4+1}{x^3}\,dx$
(c) $\int_{-7}^{1} |x+3|\,dx$

13 [2] State the second derivative test

[8] 14. Answer each question as True (T) or False (F).

(a) If f, g are functions such that, for all x, $f' = g$ and $g' = f$ the $f^2 - g^2$ is constant.

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(b) If $\int_a^b f(x)dx$ exists then f must be differentiable on $[a, b]$.

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(c) If f, g are continuous functions with $f(x) > g(x)$ then

$\int_a^b |f(x)|dx > \int_a^b |g(x)|dx.$ ________________

(d) $\lim_{x \to 0}(x^x)$ is 1. ________________

(e) $e^{3\ln 5} = 125.$ ________________

(f) If f is a continuous function on $(-\infty, \infty)$ and f is odd then $\int_{-a}^{a} f(x)dx = 0.$

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(g) If $f'(x) = \dfrac{4 - x^2}{(x^2+3)^2}$ then f has a local (= relative) maximum at $x = -2$.

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(h) If $y = f(x) = x^5 + x^3 + 1$ and $f^{-1}(x)$ is the inverse function of f then

$\dfrac{d}{dx}(f^{-1}(x)) = \dfrac{1}{5y^4 + 3y^2}$. ________________

Answers, Comments

1 $\frac{dy}{dx} = \frac{-x^2+3}{(x^2-4x+3)^2}$ 2 $\frac{dy}{dx} = \frac{2\tan x \sec^2 x (\ln x - 2) - \tan^2 x \left(\frac{1}{x}\right)}{(\ln x - 2)^2}$

3 $\frac{dy}{dx} = \frac{1}{x}$, $\frac{dy}{dx} = \cos x \; 2^{\sin x} \ln 2$ 4 $f'(2) = -12$

5 $9x + 7y - 30 = 0$. 6 Discussion: $y' = -\frac{x-2}{y}$: set this equal to $\frac{y}{x}$, use the equation of the curve to get the final answer(s): The equations are $y = -\frac{1}{\sqrt{3}}x$ and $y = \frac{1}{\sqrt{3}}x$

7 Discussion: Area $= A(x) = \frac{x^2\sqrt{3}}{4}$, $\frac{dx}{dt} = \frac{1}{2}$

When $x = 9$, $\frac{dA}{dt} = \frac{9\sqrt{3}}{4} \doteq 3.89$ in^2/sec

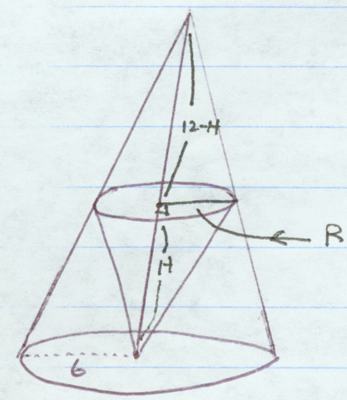
8 $x+y = 36$, $y = 36-x$, $f(x) = \sqrt{x} + \sqrt{36-x}$, $f'(x) = 0 \Rightarrow$ ek

Max $f(x)$ is $2\sqrt{18}$ occurring at $x = 18$

$0 \le x \le 36$

9 $\frac{R}{12-H} = \frac{6}{12} = \frac{1}{2}$, $R = \frac{12-H}{2}$

Volume is $\frac{\pi}{3} R^2 H$ - - - -



Max. Volume is $\frac{\pi}{3}(64)$

10 (L'HOPITAL TWICE) $\frac{1}{3}$ 11 In class

12 (a) $\frac{32}{3}$ (b) $\frac{x^2}{2} - \frac{1}{2x^2} + C$ (c) 16 13 in class

14 T F F T T T F F