UNIVERSITY OF CALGARY

DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 251 — L02 FALL 2004

MIDTERM EXAM [04-10-29(Fri)]

Total Marks =80. Duration = 50 minutes.

Work all problems. Marks are shown in brackets.

NO CALCULATORS OR FORMULA SHEETS.

PLEASE WRITE ID NUMBER ON LAST PAGE

NAME: ____

[5] 1. Find the limit

$$\lim_{x\to-\infty} \frac{x^2+x^4+x^7}{7x^7+5x^4+4} = \lim_{x\to-\infty} \frac{x^7}{7x^7} = \frac{1}{7} \leftarrow$$

[12] 2. Let $y = x^3\tan(3x) + \sin^2\left(\frac{1}{x}\right)$. Find y'.

$$y' = 3x^2\tan(3x) + 3x^3\sec^2(3x) + 2\sin\left(\frac{1}{x}\right)\cos\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right) \leftarrow$$

[5] 3. State the definition of: the function h is continuous at the point $x = -\pi$.

$$\lim_{x\to-\pi} h(x) = h(-\pi), \leftarrow$$

[15] 4. Find the equation of the tangent line to the graph of

$$y \sin(y - x) = y + x - \pi$$

at the point $(0, \pi)$.

$$y' \sin(y-x) + y\cos(y-x)(y'-1) = y'+1$$

$$-\pi(y'-1) = y'+1$$

$$y' = \frac{\pi-1}{\pi+1}$$

$$y - \pi = \frac{\pi-1}{\pi+1} x \leftarrow$$

5. Let $f(x) = x^2 + x \sin x$

[12] (a) Find the linear approximation to f at the point $x_0 = \pi$.

$$f(x_0) = \pi^2$$

$$f'(x) = 2x + x\cos x + \sin x$$

$$f'(x_0) = 2\pi - \pi = \pi$$

$$f(x) \simeq \pi^2 + \pi(x - \cancel{\pi}) \leftarrow$$

$$\simeq \pi x$$

[8] (b) Suppose the linear approximation to f at $x_0 = \pi$ is used to find approximate values of $f(x)$. What is the error if it is used at $x = \frac{\pi}{2}$?

$$f\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \pi/2$$

$$y\left(\frac{\pi}{2}\right) = \pi^2 + \pi\left(-\frac{\pi}{2}\right) = \pi^2/2$$

$$E = \left|f\left(\frac{\pi}{2}\right) - y\left(\frac{\pi}{2}\right)\right| = -\frac{\pi}{2} + \frac{\pi^2}{4} \leftarrow$$

6. For $x < 1$ the graph of certain function $f(x)$ is the curve $y = 3x^{5/3} + x$. For $x \geq 1$ the graph of $f(x)$ is the line joining the points $(1, 4)$ and $(3, y_0)$.

[8] (a) Find $\dfrac{f(1+h) - f(1)}{h}$ when $h < 0$.

$$= \frac{3(1+h)^{5/3} + 1 + h - 4}{h}$$

$$= \frac{3(1+h)^{5/3} + h - 3}{h} \leftarrow$$

[7] (b) Find $\lim\limits_{h \to 0^-} \dfrac{f(1+h) - f(1)}{h}$.

$$= \frac{d}{dx}\left(3x^{5/3} + x\right)\Big|_{x=1}$$

$$= \left(5x^{2/3} + 1\right)\Big|_{x=1}$$

$$= 6$$

[8] (c) Find a value of y_0 such that $f'(1)$ exists.

$m = \dfrac{y_0 - 4}{3 - 1} = \dfrac{y_0 - 4}{2} = NQ$ for $h > 0$

For existence of $f'(1)$ $\lim\limits_{h \to 0^-} NQ = \lim\limits_{h \to 0^+} NQ$

By (b) $\lim\limits_{h \to 0^-} NQ = 6$. So

$$6 = \lim_{h \to 0^+} \frac{y_0 - 4}{2}$$

$$6 = \frac{y_0 - 4}{2}$$

$$y_0 = 16 \leftarrow$$

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