UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 251 — L02 FALL 2004

MIDTERM EXAM [04-10-29(Fri)]

Total Marks =80. Duration = 50 minutes.
Work all problems. Marks are shown in brackets.

NO CALCULATORS OR FORMULA SHEETS.

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[15] 1. Find the equation of the tangent line to the graph of

$$y\sin(y-x) = \sin y$$

at the point (π, π).

$$y'\sin(y-x) + y\cos(y-x)(y'-1) = y'\cos y$$

$$\pi(y'-1) = -y'$$

$$y' = \frac{\pi}{\pi+1}$$

$$y-\pi = \frac{\pi}{\pi+1}(x-\pi) \leftarrow$$

[5] 2. Find the limit

$$\lim_{x\to\infty} \frac{x+x^7+3}{8x^8+7x^7+x+1} = \lim_{x\to-\infty} \frac{x^7}{8x^8} = 0 \leftarrow$$

[12] 3. Let $y = \cos^3 x + x^2 \sin\left(\frac{1}{x}\right)$. Find y'.

$$y' = 3\cos^2 x(-\sin x) + 2x\sin\left(\frac{1}{x}\right) + x^2\cos\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right) \leftarrow$$

4. Let $f(x) = 4x + \sin x + x\cos x$

[12] (a) Find the linear approximation to f at the point $x_0 = -\pi$.

$f(x_0) = -4\pi + \pi = -3\pi$

$f'(x) = 4 + 2\cos x - x\sin x$

$f'(x_0) = 4 - 2 = 2$

$f(x) \approx -3\pi + 2(x+\pi) \leftarrow$

$\approx 2x - \pi$

[8] (b) Suppose the linear approximation to f at $x_0 = \pi$ is used to find approximate values of $f(x)$. What is the error if it is used at $x = -2\pi$?

$f(\pi) = 3\pi \qquad f'(\pi) = 2$

$f(x) \approx 3\pi + 2(x-\pi) = \pi + 2x$

$f(-2\pi) = -8\pi - 2\pi = -10\pi$

$y(-2\pi) = \pi - 4\pi = -3\pi$

$E = |-10\pi - (-3\pi)| = 7\pi \leftarrow$

[5] 5. State the definition of: the function g is continuous at the point $x = -3$.

$\lim_{x \to -3} g(x) = g(-3) \leftarrow$

6. For $x < 0$ the graph of certain function $f(x)$ is the curve $y = 3(x+1)^{5/3} + 5$. For $x \geq 0$ the graph of $f(x)$ is the line joining the points $(0, 8)$ and $(6, y_0)$.

[8] (a) Find $\dfrac{f(h) - f(0)}{h}$ when $h < 0$.

$$= \frac{3(h+1)^{5/3} + 5 - 8}{h}$$

$$= \frac{3(h+1)^{5/3} - 3}{h} \leftarrow$$

[7] (b) Find $\displaystyle\lim_{h \to 0^-} \frac{f(h) - f(0)}{h}$.

$$= \frac{d}{dx}\left[3(x+1)^{5/3} + 5\right]\Big|_{x=0}$$

$$= 5(x+1)^{2/3}\Big|_{x=0}$$

$$= 5 \leftarrow$$

[8] (c) Find a value of y_0 such that $f'(0)$ exists.

$$m = \frac{y_0 - 8}{6 - 0} = NQ \text{ for } h > 0$$

$f'(0)$ exists if $\displaystyle\lim_{h \to 0^-} NQ = \lim_{h \to 0^+} NQ$

by (b) $\displaystyle\lim_{h \to 0^-} NQ = 5$. So $\displaystyle 5 = \lim_{h \to 0^+} \frac{y_0 - 8}{6}$

$$5 = \frac{y_0 - 8}{6}$$

ID NUMBER: ____________________

$$y_0 = 38 \leftarrow$$