UNIVERSITY OF CALGARY

DEPARTMENT OF MATHEMATICS AND STATISTICS

MATHEMATICS 251 — L02 FALL 2004

MIDTERM EXAM [04-10-29(Fri)]

Total Marks =80. **Duration = 50 minutes.**

Work all problems. Marks are shown in brackets.

NO CALCULATORS OR FORMULA SHEETS.

PLEASE WRITE ID NUMBER ON LAST PAGE

NAME: ______________________________

[12] 1. Let $y = x^2 \sin(3x) + \tan^2\left(\frac{1}{x}\right)$. Find y'.

$y' = 2x\sin 3x + 3x^2 \cos 3x + 2\tan\left(\frac{1}{x}\right)\sec^2\left(\frac{1}{x}\right)\left(\frac{-1}{x^2}\right) \leftarrow$

[5] 2. Find the limit

$$\lim_{x\to-\infty} \frac{x + x^5 + 7}{3x - 4x^4 - 6x^5} = \lim_{x\to-\infty} \frac{x^5}{-6x^5} = \frac{-1}{6} \nwarrow$$

[15] 3. Find the equation of the tangent line to the graph of

$$y\cos(x+y) = x + y - \frac{\pi}{2}$$

at the point $\left(0, \frac{\pi}{2}\right)$.

$y'\cos(x+y) - y\sin(x+y)(1+y') = 1+y'$

$-\frac{\pi}{2}(1+y') = 1+y'$

$y' = -1$

$y - \frac{\pi}{2} = -(x-0).$

$y = -x + \pi/2 \leftarrow$

[5] 4. State the definition of: the function f is continuous at the point $x = 7$.

$$\lim_{x \to 7} f(x) = f(7).$$

5. Let $f(x) = x^2 - x\cos x$

[12] (a) Find the linear approximation to f at the point $x_0 = \dfrac{\pi}{2}$.

$f(x_0) = \pi^2/4.$

$f'(x) = 2x + x\sin x - \cos x$

$f'(x_0) = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$

$f(x) \approx \frac{\pi^2}{4} + \frac{3\pi}{2}(x - \pi/2)$

[8] (b) Suppose the linear approximation to f at $x_0 = \frac{\pi}{2}$ is used to find approximate values of $f(x)$. What is the error if it is used at $x = \pi$?

$f(\pi) = \pi^2 + \pi$

$y(\pi) = \frac{\pi^2}{4} + \frac{3\pi^2}{4} = \pi^2$

$E = |f(\pi) - y(\pi)| = \pi$

6. For $x < 1$ the graph of certain function $f(x)$ is the curve $y = 6x^{3/2} + x$. For $x \geq 1$ the graph of $f(x)$ is the line joining the points $(1, 7)$ and $(4, y_0)$.

[8] (a) Find $\dfrac{f(1+h) - f(1)}{h}$ when $h < 0$.

$$\frac{f(1+h) - f(1)}{h} = \frac{6(1+h)^{3/2} + 1 + h - 7}{h} = \frac{6(1+h)^{3/2} + h - 6}{h}$$

[7] (b) Find $\displaystyle\lim_{h \to 0^-} \frac{f(1+h) - f(1)}{h}$.

$$= \frac{d}{dx}\left(6x^{3/2} + x\right)\Big|_{x=1}$$

$$= \left(9x^{1/2} + 1\right)\Big|_{x=1} = 10 \leftarrow$$

[8] (c) Find a value of y_0 such that $f'(1)$ exists.

$$m = \frac{y_0 - 7}{4 - 1} = \frac{y_0 - 7}{3} = \text{N.Q. for } h > 0$$

For existence of $f'(1)$ $\displaystyle\lim_{h \to 0^-} NQ = \lim_{h \to 0^+} NQ$

By (b) $\displaystyle\lim_{h \to 0^-} NQ = 10$ $\displaystyle\lim_{h \to 0^+} NQ = \lim_{h \to 0^+} \frac{y_0 - 7}{3}$

So $10 = \dfrac{y_0 - 7}{3}$

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$$y_0 = 37 \leftarrow$$