Math 251 L02 Fall 2004 Tuesday Lab.

Quiz # 2 Duration: 35 minutes

[Marks] Total marks = 30

[3] 1. State the $\epsilon-\delta$ definition of $\lim_{x \to a} f(x) = L$.

2. Consider the equation $x^3 + 2x - 22 = 0$.
Let r denote a root of the equation.

[3] (a) Find an interval of length 1 containing r.

[3] (b) Find an interval of length $\frac{1}{2}$ containing r.

[3] (c) Find an estimate for r with Error $< \frac{1}{4}$.

OVER

3. Let functions f, g, h, and k be defined by

$$f(x) = \frac{x}{x+2}, \quad g(x) = \frac{x+1}{x-1}, \quad h = f \circ g, \quad k = g \circ f.$$

[4] (a) Find D_h and write it in interval notation.

[3] (b) Find D_k and write it in interval notation.

4. Find the limits and state your answers as ∞ or $-\infty$ where appropriate.

[3] (a) $\displaystyle \lim_{x \to 3} \frac{x^2-9}{x^2-x-6}$

[2] (b) $\displaystyle \lim_{x \to -\infty} \left(x^3 - 3x^5 \right)$

[6] (c) $\displaystyle \lim_{x \to \infty} \left[\sqrt{9x^2+2x} - 3x \right]$

Math 251 L02 Fall 2004 Tues. Lab

QUIZ # 2 Solutions [Marks]

1. For every $\epsilon > 0$ there exists $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x-a| < \delta$. [3]

2. (a) $f(x) \equiv x^3 + 2x - 22$

$f(2) = -10 \quad f(3) = 11 \quad \text{IVT} \Rightarrow r \in (2,3) \leftarrow$ [1] [2]

(b) $f(5/2) = \frac{125}{8} + 5 - 22 = \frac{165-176}{8} < 0 \quad \text{IVT} \Rightarrow r \in (5/2, 3) \leftarrow$ [2] [1]

(c) $r \approx \frac{5}{2} + \frac{1}{4} = \frac{11}{4}$ = midpoint of $(5/2, 3)$ so $E < \frac{1}{4}$. [3]

3. (a) For D_h $x \in D_g$ is $x \neq 1$ $g(x) \in D_f$ is $g(x) \neq -2$, $x \neq \frac{1}{3}$ [1] [2]

$D_h = (-\infty, \frac{1}{3}) \cup (\frac{1}{3}, 1) \cup (1, \infty)$. $\leftarrow$ [1]

(b) For D_k $x \in D_f$ is $x \neq -2$. $f(x) \in D_g$ is $f(x) \neq 1$ [1]

$f(x) = \frac{x}{x+2} \Rightarrow x \neq 1$ So $D_k = (-\infty, -2) \cup (-2, \infty)$. $\leftarrow$

[1] [1]

4. (a) $\lim\limits_{x\to 3} \frac{(x-3)(x+3)}{(x-3)(x+2)} = \lim\limits_{x\to 3} \frac{x+3}{x+2} = 6/5 \leftarrow$ [2] [1]

(b) $+\infty \leftarrow$ [2]

(c) $\lim\limits_{x\to\infty} \frac{(\sqrt{9x^2+2x} - 3x)(\sqrt{9x^2+2x} + 3x)}{\sqrt{9x^2+2x} + 3x}$ [2] $= \lim\limits_{x\to\infty} \frac{2x}{\sqrt{9x^2+2x} + 3x}$

$= \lim\limits_{x\to\infty} \frac{x}{x} \frac{2}{\sqrt{9 + \frac{2}{x}} + 3} = \frac{1}{3} \leftarrow$ [1]

[3]

Math251 L02 Fall 2004 Thurs. Lab.

Quiz # 2 Duration: 35 minutes

[Marks] Total marks = 30

[3]. 1 State the ϵ-δ definition of $\lim_{x \to a} f(x) = L$.

2. Consider the equation $x^3 + 2x - 16 = 0$.
Let r denote a root of the equation.

[3] (a) Find an interval of length 1 containing r.

[3] (b) Find an interval of length $\frac{1}{2}$ containing r.

[3] (c) Find an estimate for r with Error $< \frac{1}{4}$.

OVER

3. Let functions f, g, h, and k be defined by

$$f(x) = \frac{x}{x+3} \quad g(x) = \frac{x}{x-1} \quad h = f \circ g \quad k = g \circ f.$$

[4] (a) Find D_h and write it in interval notation.

[3] (b) Find D_k and write it in interval notation.

4. Find the limits and state your answers as ∞ or $-\infty$ where appropriate.

[3] (a) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x^2 - 4}$

[2] (b) $\lim\limits_{x \to -\infty} \dfrac{x - 5x^3}{-x^2 + 1}$

[6] (c) $\lim\limits_{x \to \infty} \left[\sqrt{4x^2 + 3x} - 2x\right]$

Math 251 L02 Fall 2004 Thurs. Lab.
Quiz #2 Solutions.

1. For every $\epsilon > 0$ there exists $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x-a| < \delta$.

2. (a) Let $f(x) = x^3 + 2x - 16$.
$f(2) = -4 \quad f(3) = 17 \quad IVT \Rightarrow r \in (2,3) \leftarrow$

(b) $f(5/2) = \dfrac{125 + 40 - 128}{8} > 0$. $IVT \Rightarrow r \in (2, 5/2) \leftarrow$

(c) $r \approx 2 + \frac{1}{4} = \frac{9}{4}$ = midpt. of $(2, 5/2)$ So Error $< \frac{1}{4}$.

3. (a) $x \in D_g \Rightarrow x \neq 1 \quad g(x) \in D_f \Rightarrow x \neq \frac{3}{4}$.
$D_h = (-\infty, 3/4) \cup (3/4, 1) \cup (1, \infty)$. $\leftarrow$

(b) $x \in D_f \Rightarrow x \neq -3$. $f(x) \in D_g \Rightarrow f(x) \neq 1 \neq \frac{x}{x+3}$ holds for all x.
So $D_k = (-\infty, -3) \cup (-3, \infty)$. $\leftarrow$

4. (a) $\displaystyle\lim_{x\to 2} \frac{(x+3)(x-2)}{(x+2)(x-2)} = \frac{5}{4} \leftarrow$

(b) $\displaystyle\lim_{x\to -\infty} \left(\frac{-5x^3}{-x^2}\right) = -\infty$

(c) $\displaystyle\lim_{x\to\infty} \frac{\left[\sqrt{4x^2+3x} - 2x\right]\left[\sqrt{4x^2+3x} + 2x\right]}{\sqrt{4x^2+3x} + 2x} = \lim_{x\to\infty} \frac{3x}{\sqrt{4x^2+3x} + 2x}$

$\displaystyle = \lim_{x\to\infty} \frac{x}{x} \frac{3}{\sqrt{4 + \frac{3}{x}} + 2} = 3/4 \leftarrow$