Math 251 L02 Fall 2004 Tuesday Lab.

Quiz # 3 Duration: 35 minutes

[Marks] Total marks = 30

[6] 1. Find the value of the limit

$$\lim_{x \to 0} \frac{1-\cos(3x)}{x \sin(3x)}$$

2. Let $f(x) = \sqrt{2x+1}$.

[7] (a) Use the limit definition of derivative to find $f'(x)$.

[3] (b) Find the equation of the tangent line to the graph of f at $x=4$.

[7] 3. Let $f(x) = \dfrac{\sin x + \cos x}{1+\cos x}$

Find $f'(x)$ and simplify your answer.

[7] 4. Let $f(x) = x + 2\sqrt{x}$.

Find a point on the graph of f where the tangent line to f is parallel to the secant line which cuts the curve at $x=1$ and $x=9$.

Math 251 L02 Fall 2004 Tues. Lab
Quiz # 3 Solutions

1. $\lim_{x\to 0} \frac{1-\cos^2 3x}{x\sin 3x\,(1+\cos 3x)} = \lim_{x\to 0} \frac{\sin^2 3x}{x\sin 3x\,(1+\cos 3x)}$ [2]

$= \lim_{x\to 0} 3\,\frac{\sin 3x}{3x\,(1+\cos 3x)}$ [2] $= 3\cdot 1\cdot \frac{1}{2} = \frac{3}{2}$ ← [2]

2 (a) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h}$ [1] [1]

$= \lim_{h\to 0} \frac{2(x+h)+1-(2x+1)}{h(\sqrt{\quad}+\sqrt{\quad})}$ = [3]

$= \lim_{h\to 0} \frac{2h}{h(\sqrt{\quad}+\sqrt{\quad})} = \frac{2}{\sqrt{2x+1}+\sqrt{2x+1}}$ [2]

$f'(x) = \frac{1}{\sqrt{2x+1}}$ ←

(b) $f'(4) = \frac{1}{\sqrt{9}} = \frac{1}{3}$ [1] $y-3 = \frac{1}{3}(x-4)$ ← [2]

3. $f' = \frac{(1+c)(c-s)-(s+c)(-s)}{(1+c)^2} = \frac{c-s+c^2+s^2}{(1+c)^2} = \frac{1+c-s}{(1+c)^2}$ ←

[4] [1] [2]

4. At $x=1$ $y=f(1)=3$

At $x=9$ $y=f(9)=15$

$m = \frac{15-3}{9-1} = \frac{3}{2}$ [2]

$f' = 1+\frac{1}{\sqrt{x}}$ [2] $1+\frac{1}{\sqrt{x}} = \frac{3}{2}$ [2] $x=4$

$(4,8)$ ← [1]

Math251 L02 Fall 2004 Thurs. Lab.

Quiz # 3 Duration: 35 minutes

[Marks] Total marks = 30

1. Find the value of the limit.

$$\lim_{x \to 0} \frac{x \tan x}{1 - \cos x}$$

2. Let $f(x) = \sqrt{1-x}$

(a) Use the limit definition of derivative to find $f'(x)$.

(b) Find the equation of the tangent line to the graph of f at $x = -3$.

3. Let $$f(x) = \frac{\cos x + \sin x}{1 - \sin x}$$

Find $f'(x)$ and simplify your answer.

4. Let $f(x) = x + \frac{12}{x^2}$

Find a point on the graph of f where the tangent line to f is parallel to the secant line which cuts the curve at $x = 1$ and $x = 2$

Math 251 L02 Fall 2004 Thurs. Lab.

Quiz # 3 Solutions

same marking scheme as Tues. Labs

Answers

1. $\lim_{x\to 0} \frac{x\,s}{c(1-c)} = \lim_{x\to 0} \frac{x\,s(1+c)}{c\,s^2} = \lim_{x\to 0} \frac{x}{s}\,\frac{1+c}{c} = 2.$ ←

2. (a) $f'(x) = \lim_{h\to 0} \frac{\sqrt{1-(x+h)} - \sqrt{1-x}}{h} = \frac{-1}{2\sqrt{1-x}}$ ←

(b) $f'(-3) = -\frac{1}{4} \qquad f(-3) = 2$

$y - 2 = -\frac{1}{4}(x+3)$ ←

3. $f'(x) = \frac{\cos x + 1 - \sin x}{(1-\sin x)^2}$ ←

4. $(1, 13)$
$(2, 5)$ $\qquad m = -8$

$f' = 1 - \frac{24}{x^3} = -8$

$x^3 = \frac{8}{3} \qquad x = \frac{2}{\sqrt[3]{3}}.$ ←