Math 251 LO2 Fall 2004 Thurs. Lab.

Quiz # 4   Duration: 35 minutes

[Marks]   Total marks = 30

[4] 1. Find the limit. $\lim\limits_{x \to 0^+} \dfrac{\ln|\tan x|}{\ln(\sin x)}$,

2. Given $f(x) = \dfrac{-1}{x(x-1)}$, $f'(x) = \dfrac{2x-1}{x^2(x-1)^2}$, $f'' = \dfrac{-2(3x^2-3x+1)}{x^3(x-1)^3}$.

[3] (a) Find all vertical and horizontal asymptotes of f.

[2] (b) Find all intervals where f is increasing or decreasing.

[2] (c) Find the (x,y) coordinates of all relative extrema of f.

[3] (d) Find all intervals where f is concave up or concave down.

[8] (e) Sketch the graph of f.

[8] 3. Given $f(x) = (x-2)e^{-x}$, $f'(x) = -(x-3)e^{-x}$, $f''(x) = (x-4)e^{-x}$.

Sketch the graph of f. Label all points on the graph which are relative extrema or inflection points.

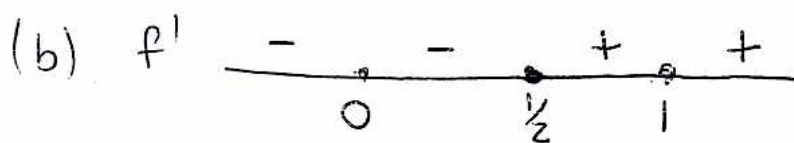1. $\ln(\tan x) \to -\infty$ $\ln(\sin x) \to -\infty$

$$\frac{\frac{d}{dx} \ln \tan x}{\frac{d}{dx} \ln \sin x} = \frac{\frac{1}{\tan x} \frac{1}{\cos^2 x}}{\frac{1}{\sin x} \cos x} = \frac{\sin x}{\frac{\sin x}{\cos x} \cos^3 x}$$

$$= \frac{1}{\cos^2 x}$$

$$\lim_{x \to 0^+} \frac{1}{\cos^2 x} = 1 \quad \leftarrow$$
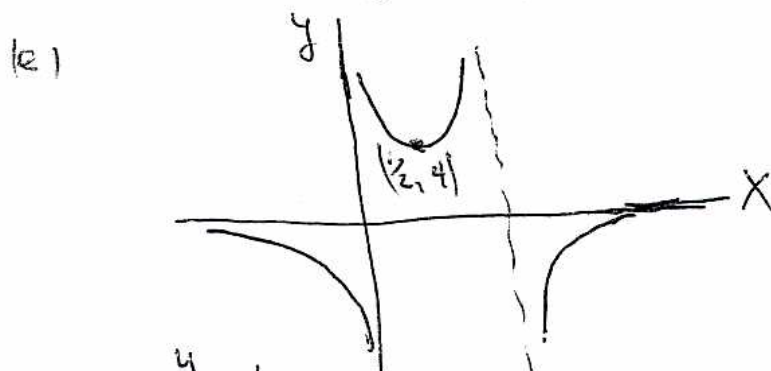
2. (a) V.A. $x = 0$ & $x = 1$,   H.A. $y = 0$

(b) f'

$\underset{\underset{0}{}}{-} \quad \underset{\underset{\frac{1}{2}}{}}{-} \quad \underset{\underset{1}{}}{+} \quad +$

Inc. $\left[\frac{1}{2}, 1\right)$ & $(1, \infty)$ Dec. $(-\infty, 0)$ & $\left(0, \frac{1}{2}\right]$

(c) $\left(\frac{1}{2}, 4\right)$ Rel. min.

(d) f''

$\underset{\underset{0}{}}{-} \quad \underset{\underset{1}{}}{+} \quad -$

C. up. $(0, 1)$

C. down. $(-\infty, 0)$ & $(1, \infty)$

(e)

$\left(\frac{1}{2}, 4\right)$

3.

$\left(3, \frac{1}{e^3}\right)$

$\left(4, \frac{2}{e^4}\right)$