Math 251 L02 Fall 2004 Tuesday Lab.

Quiz # 5 Duration: 35 minutes

[Marks] Total marks = 30

1. Suppose the position $s(t)$ of a moving body is given by

$$s(t) = t e^{-t} \quad \text{for } t \geq 0$$

where t is the time.

[6] (a) Find the velocity $v(t)$ and the acceleration $a(t)$.

[6] (b) Find all intervals where the body is speeding up and all intervals where it is slowing down.

2. Find the integrals.

[5] (a) $\int_1^2 \left[1 + \frac{1}{x} + \frac{1}{x^2}\right] dx$

[5] (b) $\int \frac{\cos\left(\frac{1}{x}\right)}{x^2}\, dx$

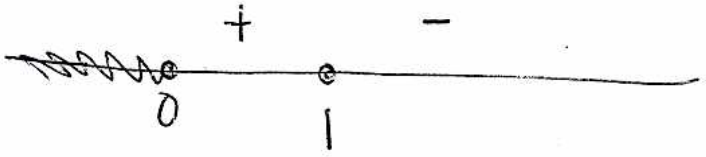
[8] (c) $\int \frac{3x^5}{\sqrt{x^3+1}}\, dx$

Math 251 L02 Fall 2004 Tues. Lab
Quiz #5 Solutions [10 Marks]

1. (a) $v = \frac{ds}{dt} = -e^{-t}(t-1)$ [3]

$a = \frac{dv}{dt} = (t-2)e^{-t}$ [3]

(b) signs v



signs a

speeding up $(1,2)$ [2]

slowing down $(0,1)$ and $(2,\infty)$ [2] [2]

2. (a) $\left[x + \ln x - \frac{1}{x}\right]_1^2 = \frac{3}{2} + \ln 2 \leftarrow$ [1,1,1] [2]

(b) $u = \frac{1}{x}$ $du = -\frac{dx}{x^2}$ [2]

$\int \frac{\cos(\frac{1}{x})}{x^2}\,dx = \int \frac{\cos u}{x^2}(-x^2)\,du = -\int \cos u\,du = -\sin u + c$ [3]

$= -\sin\frac{1}{x} + c. \leftarrow$

(c) $u = x^3 + 1$ $du = 3x^2\,dx$ [2]

$\int \frac{3x^5}{\sqrt{x^3+1}}\,dx = \int \frac{3x^5}{\sqrt{u}}\,\frac{du}{3x^2} = \int \frac{x^3}{\sqrt{u}}\,du = \int \frac{u-1}{\sqrt{u}}\,du$

$= \int (u^{1/2} - u^{-1/2})\,du = \frac{2}{3}u^{3/2} - 2u^{1/2} + c$ [3] [2]

$= \frac{2}{3}(x^3+1)^{3/2} - 2(x^3+1)^{1/2} + c \leftarrow$ [1]