MATHEMATICS 251 FINAL FALL 2002

SECTION L04/L13 TIME: 2 HOURS

Calculators are *not* allowed on this exam.

NAME _____

I.D. NUMBER _____

1. Find the equations of the two tangent lines to the circle

$$x^2 + 4x + y^2 + 3 = 0$$

that pass through the origin. [10]

2. The hour and minute hands of a clock are six and eight centimeters long, respectively. How fast are the tips of the hands separating at 12:20? [10]

3. Of all right circular cylinders with a given surface area find the one with maximum volume. Note: The ends of the cylinders are closed. [10]

4. From what height above the earth must a ball be dropped in order to strike the ground with a speed of 136 feet per second? Recall that the acceleration due to gravity is 32 ft/sec^2. [10]

5. Let

$$f(x) = \frac{ax + b}{cx + d}.$$

What condition on a, b, c and d will make f equal to its inverse function f^{-1}? [10]

6. Recall that the natural logarithm of a positive number x may be defined as

$$\ln x = \int_1^x \frac{1}{t}\, dt.$$

Show that

(a)

$$\int_1^{ab} \frac{1}{t}\, dt = \int_1^a \frac{1}{t}\, dt + \int_1^b \frac{1}{t}\, dt$$

and hence conclude that $\ln(ab) = \ln a + \ln b$.

(b) Prove the addition law for the exponential function

$$e^{a+b} = e^a e^b.$$

Hint: you can use the result of the previous section and the fact that the natural logarithm and the exponential function are inverse functions. [10]