

1. No work need be shown for the following quick answer questions. (2 points each).

- (a) Compute the following limit if it exists

$$\lim_{x \rightarrow 0^+} \sqrt{x}(\ln x)^2$$

ANSWER _____

- (b) Find $f'(1)$ if $f(x) = e^x \ln \sqrt{x}$.

ANSWER _____

- (c) $f'(\psi)$ if $f(\theta) = \cot \theta + 2 \sec \theta$ and $\sin \psi = 3/4$.

ANSWER _____

- (d) Evaluate

$$\int_1^2 (2-x)^5 dx$$

ANSWER _____

- (e) Evaluate

$$\int_2^7 \frac{1}{3x+2} dx$$

ANSWER _____

2. Do only one of the following two problems

(a) Find the equation of the line tangent to the devil's curve

$$y^4 - 4y^2 + x^4 - 9x^2 = 0$$

at the point $(x, y) = (0, 2)$. (6 points)

(b) Find the values of k and r that make the circle $x^2 + (y - k)^2 = r^2$ tangent to the parabola $y = x^2$ at the point $(0, 0)$ and that also make the second derivatives d^2y/dx^2 have the same value on both curves there. Circles like this one that are tangent to a curve and have the same second derivatives there are called osculating circles (from the Latin *osculari* meaning 'to kiss'). (10 points)

3. At what rate is the area of an equilateral triangle increasing if a side is 10 cm long and increasing at a rate of 0.5 cm/s? (Assume that the triangle stays equilateral during the expansion) (8 points)

4. If you average 30 km/h on a 150 km trip and then return over the same 150 km at a rate of 50 km/h, what is your average speed for the trip? (6 points)

5. Do only one of the following two problems
- (a) The sum of two nonnegative numbers is $\sqrt{41}/3$. Find the numbers so that the sum of twice the square root of one number and three times the square root of the other is as large as possible. (7 points)
 - (b) A right circular cone of height 12 and base radius 6 contains another cone upside down within it. The two bases are parallel, and the vertex of the smaller cone lies at the center of the larger cone's base. What are the dimensions of the smaller cone that will give it the largest possible volume? (10 points)

6. Sketch the graph of $y = x^2/(2 + x^2)$. Be sure to label all important qualitative features, such as asymptotes, critical points, and inflection points (if any). (10 points)

7. Explain how you would find the value of the smallest positive solution to the equation $\sin 3x = 3 \cos x$. (7 points)