MATHEMATICS 251 MIDTERM

NAME _____

1. If $f(x) = (x-2)/(x+1)$ and $g(x) = (2+x)/(1-x)$, compute the composite $f^{-1} \circ g \circ f(2)$. [5]

2. Let $f(x)$ be defined by

$$f(x) = \begin{cases} x+2 & x < 1 \\ ax^2 + bx & x \geq 1 \end{cases}$$

Find a and b so that not only f, but f' is also continuous. [5]

3. Compute the following limit.

$$\lim_{h \to 0^+} \frac{1}{h} \left\{ \tan\left(h + \frac{\pi}{6}\right) - \frac{1}{\sqrt{3}} \right\}$$

Clue: It is probably easier *not* to use the addition formula for the tangent function. [5]

4. Find the area of the triangle formed by the tangent line to $xy = 1$ at (a, a^{-1}) and the x and y axes. (This is a theorem of Archimedes). [5]

5. Do only one of the following two problems.

   (a) If the third-life (time for $\frac{1}{3}$ of a sample to decay) of the radioactive substance meredithium is 1000 seconds, what is the half-life of meredithium? [6]

   (b) An alert hiker sees that a boulder falls off a cliff and notices that it falls the last third of the way in 3/2 seconds. What is the height of the cliff? You may assume the boulder falls straight down with an initial velocity of zero, and that the acceleration due to gravity is $10m/s^2$. [10]