

Math 251, Fall 2004
PRACTICE PROBLEMS

1. Express the function $f(x)$ in piecewise form without using absolute values $f(x) = 3|x - 2| - |x + 1|$

2. Find or simplify $\frac{f(x+h) - f(x)}{h}$ where $h \neq 0$ and $f(x) = 4x^2 - 5x + 7$.

3. Find the domain and the range of the function $g(x)$.

(a) $g(x) = \sqrt{x^2 - x - 2}$

4. Find the value of the constant k , if possible, that will make the function continuous everywhere.

$$f(x) = \begin{cases} kx^2 & x \leq 2 \\ 2x + k & x > 2 \end{cases}$$

5. State the following Theorems:

- (a) Intermediate-Value Theorem
- (b) Mean-Value Theorem
- (c) Fundamental Theorem of Calculus.

6. What is the definition of the derivative?

7. Show that $f(x) = 5 - x - x^2$ has at least one solution on the interval $[-4, 0]$.

8. Find the following limits

(a) $\lim_{x \rightarrow \infty} \frac{3x + 4}{\sqrt{2x^2 - 5}}$

(b) $\lim_{x \rightarrow 0} \frac{3x^2}{1 - \cos^2(\frac{1}{2}x)}$

(c) $\lim_{h \rightarrow \frac{\pi}{2}} \frac{1 - \sin h}{\frac{1}{2}\pi - h}$ (Hint use $t = \frac{\pi}{2} - h$)

9. Find $\frac{dy}{dx}$ by implicit differentiation.

$$x \sin y + y \cos x = 1$$

10. Find $\frac{dy}{dx}$ for the following:

(a) $y = \sqrt{3}^{x \sin x}$

(b) $y = (\tan x)^{\ln x}$

11. At which points of the curve $xy = (1 - x - y)^2$ the tangent line is parallel to the x -axis?
12. Use local linear approximation to estimate $\cos(46^\circ)$.
13. Solve for x the following equation $\ln\left(\frac{1}{x^2}\right) + \ln(4x^6) = \ln 4$
14. Find the critical points of the function $f(x) = x^3 - 3x^2 + 3$ and determine if they are maximum, minimum or none.
15. Find the intervals where the function $f(x) = xe^{2x}$ is increasing and decreasing.
16. Find the intervals where the function $f(x) = x^2 \ln x$ is concave up and concave down and determine the inflection points.
17. A rancher has 200feet of fencing with which he wants to enclose two adjacent regular corrals. What dimensions should be used so that the enclose area will be a maximum?
18. Solve the following integrals:
 - (a) $\int \sin(2x) \cos(2x) dx$
 - (b) $\int \frac{\csc^2 x}{\cot^3 x} dx$
 - (c) $\int_{\pi/2}^{2\pi/3} \sec^2\left(\frac{x}{2}\right) dx$
 - (d) $\int_1^3 \frac{e^{\frac{3}{x}}}{x^2} dx$
 - (e) $\frac{d}{dx} \int_x^0 \frac{t}{\cos t} dt$

Solutions

1. $f(x) = \begin{cases} -2x + 7 & \text{if } x < -1 \\ -4x + 5 & \text{if } -1 \leq x < 2 \\ 2x - 7 & \text{if } x \geq 2 \end{cases}$
2. $8x + 4h - 5$
3. Domain $(-\infty, -1] \cup [2, \infty)$
Range $y \geq 0$.
4. $k = 4/3$
5. .
6. .
7. .
8. (a) $\frac{3}{\sqrt{2}}$

(b) 12

(c) 0

9. $\frac{dy}{dx} = \frac{y \sin x - \sin y}{x \cos y + \cos x}$

10. (a) $\frac{dy}{dx} = (\ln \sqrt{3} [\sin x + x \cos x]) \sqrt{3}^{x \sin x}$

(b) $\frac{dy}{dx} = \left(\frac{\ln(\tan x)}{x} + \frac{\ln x}{\tan x} \sec^2 x \right) (\tan x)^{\ln x}$

11. $y = 2 - 2x$

12. $\cos(46^\circ) \approx 1/\sqrt{2} \left(1 - \frac{\pi}{180}\right)$.

13. $x = \pm 1$

14. 0 is a maximum and 2 is a minimum.

15. $f(x)$ is increasing on $(-\frac{1}{2}, \infty)$ and decreasing on $(-\infty, -\frac{1}{2})$

16. $f(x)$ is concave up on $(e^{-\frac{3}{2}}, \infty)$ and concave down on $(-\infty, e^{-\frac{3}{2}})$. $x = e^{-\frac{3}{2}}$ is an inflection point.

17. Dimensions: 50feet by 100/3feet.

18. (a) $\frac{\sin^2(2x)}{4} + c$

(b) $\frac{1}{2 \cot^2 x} + c$

(c) $2\sqrt{3} - 2$

(d) $-\frac{1}{3}(e - e^3)$

(e) $-\frac{x}{\cos x}$