

Math 251, Fall 2004
Worksheet

1. Solve the following inequalities

(a) $\frac{2}{5-x} \geq \frac{4}{x}$

(b) $(x-4)(x+4) > 0$

2. Find the domain and the range of

(a) $f(x) = \frac{x+2}{x-1}$

3. Find the domain of

$$f(x) = \sqrt{\frac{2}{x^2+3}}$$

4. Graph the function $\sin(\frac{1}{4}x) + 1$

5. Find the following limits

(a) $\lim_{x \rightarrow \infty} \sqrt[3]{\frac{5x+4}{2x-4}}$

(b) $\lim_{x \rightarrow 0^+} \frac{\sin 3x}{1 - \cos 6x}$

(c) $\lim_{h \rightarrow 0} \frac{\tan 4h}{\sin 8h}$

(d) $\lim_{x \rightarrow 1^-} \frac{2x^2 + 3x + 1}{|2x^2 + 3x + 1|}$

6. Let $f(x) = \begin{cases} -x^4 + 3 & \text{if } x \leq 2 \\ x^2 + 9 & \text{if } x > 2 \end{cases}$. Is f continuous everywhere? Justify your conclusion.

7. Use intermediate value Theorem to show that the equation $x^3 - 4x + 2 = 0$ has a solution on the interval $[1, 2]$.

8. Find the points of discontinuity if any of the following functions

(a) $y = \frac{x-4}{x^2-16}$

(b) $y = \frac{\sin \theta}{\theta}$

(c) $y = \sqrt[3]{3x-1}$

9. Find $\frac{dy}{dx}$ of the following

(a) $\frac{1}{y} + \frac{1}{2x} = 1$

(b) $y = (\sin(x^2))^{-\frac{1}{2}}$

(c) $y = \tan(1 + \sin(x^3))$

(d) $\sin(x^3y^3) = x^2 + x$

10. Two cyclists start moving from the same point. One travels south at 50km/h and the other travels west at 30km/h. At what rate is the distance between the two cyclists increasing two hours later?
11. At which points of the curve $xy = (1 - x - y)^2$ the tangent line is parallel to the x -axis?
12. Use local linear approximation to estimate $\cos(46^\circ)$.
13. Solve for x the following equation $\ln\left(\frac{1}{x^2}\right) + \ln(4x^6) = \ln 4$

Solutions

1. **a.** $(-\infty, 0) \cup \left[\frac{10}{3}, 5\right)$ **b.** $(-\infty, -4) \cup (4, \infty)$
2. Domain $\{x \mid x \neq 1\}$, Range $\{y \mid y \neq 1\}$
3. $(-\infty, \infty)$
- 4.
5. **a.** $\sqrt[3]{\frac{5}{2}}$ **b.** ∞ **c.** $\frac{1}{2}$ **d.** 1
6. No continuous
- 7.
8. **a.** $x = -4, 4$ **b.** $\theta = 0$, **c.** No points of discontinuity.
9. **a.** $\frac{dy}{dx} = -\frac{y}{2x^2}$ **b.** $\frac{dy}{dx} = -x(\sin x^2)^{-\frac{3}{2}} \cos(x^2)$.
c. $\frac{dy}{dx} = \sec^2(1 + \sin(x^3)) (\cos(x^3)) (3x^2)$
d. $\frac{dy}{dx} = \frac{2x + 1 - 3 \cos(x^3 y^3) x^2 y^3}{3x^3 y^2 \cos(x^3 y^3)}$
10. $\frac{6800}{\sqrt{13600}} km/h$.
11. $y = 2 - 2x$
12. $\cos(46^\circ) \approx 1/\sqrt{2} \left(1 - \frac{\pi}{180}\right)$.
13. $x = \pm 1$