

2006 Fall

MATH 251 (L06).

Final Review Problems.

1. Evaluate (i) $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-1} - \sqrt{x^2+x}}{3x}$ (ii) $\lim_{x \rightarrow 0^-} \frac{1}{x} + \sqrt{\frac{1-2x}{x^2}}$

(iii) $\lim_{x \rightarrow 0} \frac{x}{\sin x - \ln(1+x)}$

2. Find the values of c such that the function

$$f(x) = \begin{cases} (c-b) & \text{for } x \geq -1 \\ c^2 x^2 & \text{for } x < -1 \end{cases}$$

is continuous at $x = -1$.

3. Evaluate y'' where (i) $y = (\sin x)^{\cos x}$, (ii) $y = e^x \cos 2x$,
(iii) $y = x^{\ln 3x}$, (iv) $e^y = x^3 \ln(\sin x)$.

4. Sketch the graph of (i) $y = \ln(1-x)$, (ii) $y = 3 \sin(x - \frac{\pi}{3})$,
(iii) $y = \frac{3x}{1-x^2}$.

5. Sketch the graph of the functions (giving domain, asymptotes, intervals of concave up + down, and local max/min points (if any)):

(i) $y = \frac{x-1}{x^2-4}$ (ii) $y = \frac{x}{1-4x^2}$ (iii) $y = 1 - \frac{1}{(1-x)(2+x)}$.

6. Evaluate

(i) $\int 4 - \frac{1}{x^3} + \frac{1}{4x} - \cos 3x + \tan 2x \, dx$

(ii) $\int (\sin 2x + \cos 2x)^2 \, dx$.

(iii) $\int \frac{e^x - e^{-x}}{e^x + e^{-x}} \, dx$

7. Evaluate

$$(i) \int (e^{2x} + e^{-2x})^2 dx$$

$$(ii) \int e^{\sin x} \cdot \cos x dx.$$

$$(iii) \int \cos 3t \sin^2 3t dt$$

$$(iv) \int t(1+t)^{1/3} dt$$

8. Use the fundamental Theorem of Calculus to find $\frac{dy}{dx}$ where

$$(i) y = \int_1^{x^2} (1-x^2)^{1/3} dx.$$

$$(ii) y = \int_x^{x^2} (1-\sin t)^{1/2} dt$$

$$(iii) y = \int_{-x}^{\tan x} (1-\cos 2u)^{1/4} du$$

9. Find the area bounded by the curves $y=3-x^2$ and $y=2x$. (First sketch the area).

Good Luck!