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MATH 251 (L06)

MID-TERM REVIEW - PROBLEMS.

1. Solve (a) $\frac{5x+24}{x} \geq x$ (b). $\frac{24}{|x-10|} < x$.

2. Find (a) $\lim_{x \rightarrow 0^-} \frac{1}{x} + \sqrt{\frac{1}{x^2} - \frac{12}{x}}$ (b) $\lim_{x \rightarrow \infty} \sqrt[3]{x^3 + 3x^2} - x$.

(c) $\lim_{x \rightarrow -\infty} 3x - \sqrt{9x^2 - 2x}$

3. Find a value of a which makes the function

$$f(x) = \begin{cases} \frac{|x+4|}{x+4} & \text{if } x < -4 \\ (4+a)x & \text{if } x \geq -4. \end{cases}$$

continuous.

Sketch the graph of $f(x)$ for this value of a .

4. For the function

$$y = \frac{x}{(1-x)(2+x)}$$

find (i) its vertical and horizontal asymptotes

(ii) $y'(2)$ and $y''(2)$.

(iii) the equation of the tangent line to its curve at $x=2$.

(iv) the points (if any) where its slope is 0.

5. Use definition of derivative to find the derivative of

(a) $y = \frac{3x}{1-x^2}$

(b) $y = \frac{x+1}{2x-1}$

6. Find (a) $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin \frac{x}{2}}$ (b) $\lim_{t \rightarrow 0} \frac{2t + \tan t}{3\sin t}$

7. If $\cos x = \frac{5}{13}$ and $\pi < x < 2\pi$, find (a) $\sin 2x$ (b) $\cos 2x$ (c) $\tan 2x$.

8. Find y' if (a) $y = \left(1 + \frac{3}{x} + 2x\right)^{1/2} \sin^2\left(1 + \frac{2}{x}\right)$

(b) $y = \cos^2 x \sin\left(x^2 - \frac{\pi}{4}x\right)$

(c) $y = \frac{6x}{\cos(x^2 + 3x)}$

(d) $\sin(x^2 y) + yx^2 = \cos y$.

(e) $\tan(xy + y^2) = y^2 - x$.

9. If $x^2 + xy^2 = 10$ and $\frac{dx}{dt} = 2$,

find $\frac{dy}{dt}$ at (a) $x = 3$;

(b) $y = 4$