Practice Problems for Quiz #1

P1 Solve the following inequalities for $x \in \mathbb{R}$. Write your answers in interval notations.

1.
$$(3+2x)^2 \leq 4$$

2.
$$3-2x < |3x-2|$$

3.
$$\frac{2x-1}{x+1} \geq \frac{2x+1}{x-1}$$

4.
$$\frac{1}{|x-1|} < 2$$

P2 Find the equation of the line which passes through the point $(3,2)$ and is

(a) parallel,

(b) perpendicular

to the line $2y+3x+6=0$.

Solutions

1. $(3+2x)^2 \le 4 \iff (3+2x)^2 - 4 \le 0$

$$\iff (3+2x-2)(3+2x+2) \le 0$$

$$\iff (1+2x)(5+2x) \le 0$$

$$\iff x \in [-5/2, -1/2].$$

2. $3-2x < |3x-2|$

$$|3x-2| = \begin{cases} 3x-2 & \text{if } 3x-2 \geq 0 \iff x \in [2/3, +\infty) \\ -3x+2 & \text{if } 3x-2 < 0 \iff x \in (-\infty, 2/3) \end{cases}$$

- If $x \geq 2/3$, then $|3x-2| = 3x-2$. We have

$$3-2x < 3x-2 \iff 5 < 5x \iff 1 < x \iff x \in (1, +\infty)$$

So, $I_1 = [2/3, +\infty) \cap (1, +\infty) = (1, +\infty)$

- If $x < 2/3$, then $|3x-2| = -3x+2$. We have

$$3-2x < -3x+2 \iff x < -1 \iff x \in (-\infty, -1)$$

So $I_2 = (-\infty, 2/3) \cap (-\infty, -1) = (-\infty, -1)$

Finally, the solution set of the inequality is

$$S = I_1 \cup I_2 = (1, +\infty) \cup (-\infty, -1)$$

3. The inequality $\frac{2x-1}{x+1} \geqslant \frac{2x+1}{x-1}$ is defined for $x \in \mathbb{R} \setminus \{1, -1\}$.

For $x \in \mathbb{R} \setminus \{1, -1\}$, we have

$$\frac{2x-1}{x+1} \geqslant \frac{2x+1}{x-1} \iff \frac{2x-1}{x+1} - \frac{2x+1}{x-1} \geqslant 0$$

$$\iff \frac{(2x-1)(x-1)-(2x+1)(x+1)}{(x+1)(x-1)} \geqslant 0$$

$$\iff \frac{2x^2-2x-x+1-(2x^2+2x+x+1)}{(x+1)(x-1)} \geqslant 0$$

$$\iff \frac{-6x}{(x+1)(x-1)} \geqslant 0 \iff \frac{6x}{(x+1)(x-1)} \leq 0$$

	$-\infty$		-1		0		1		$+\infty$
x		$-$		$-$	0	$+$		$+$	
$x+1$		$-$	0	$+$		$+$		$+$	
$x-1$		$-$		$-$		$-$	0	$+$	
$\frac{6x}{(x+1)(x-1)}$		$-$	$\Vert$	$+$	0	$-$	$\Vert$	$+$	

So, $S = (-\infty, -1) \cup [0, 1)$

4. The inequality $\frac{1}{|x-1|} < 2$ is defined for all x s.t. $|x-1| \neq 1$, i.e. $x \neq 1$.

For $x \neq 1$, it is equivalent to

$$\frac{1}{2} < |x-1|.$$

$$\Leftrightarrow \quad x-1 \leq -\frac{1}{2} \quad \text{or} \quad x-1 > \frac{1}{2}$$

$$\Leftrightarrow \quad x < 1/2 \quad \text{or} \quad x > \frac{3}{2}.$$

So, $S = (-\infty, 1/2) \cup (3/2, +\infty)$

P.2. The line $2y+3x+6=0$ has slope

$m = -3/2$.

(a) The line throug $(3,2)$ (parallel to the line) has equation:

$$y-2 = -3/2(x-3) \quad \text{or}$$

$$y = -\frac{3}{2}x + \frac{13}{2}.$$

(b) Any perpendicular line to $2y+3x+6=0$

has slope $m' = \frac{1}{m} = \frac{2}{3}$.

So $y-2 = \frac{2}{3}(x-3)$

$\Leftrightarrow$ $\boxed{y = \frac{2}{3}x}$ passes through $(3,2)$ and

is $\perp$ to $2y+3x+6=0$.