Practice Problems S2

P. 1. Find the equation of the line which passes through the point (3, 2) and is (a) parallel; (b) perpendicular to the line $2y+3x+6=0$.

P.2. Find the domain of the function

$$f(x)=\sqrt{\frac{x^2-4}{x-4}}.$$

P3. Write the following function in piecewise form: $f(x)=3+|2x-5|$

P.4: Find a formula for fog and specify its domain if

$$f(x)=\frac{1+x}{1-x};$$

$$g(x)=\frac{x}{1-x}.$$

Solutions

P.1. See Practice Problems for Q1.

P.2. The function $f(x) = \sqrt{\dfrac{x^2-4}{x-4}}$ is defined for $\dfrac{x^2-4}{x-4} \geq 0$ and $x-4 \neq 0$.

x	$-\infty$		-2		2		4		$+\infty$
$x+2$		$-$	0	$+$		$+$		$+$	
$x-2$		$-$		$-$	0	$+$		$+$	
$x-4$		$-$		$-$		$-$	0	$+$	
$\frac{x^2-4}{x-4}$		$-$	0	$+$	0	$-$	$\|$	$+$	

$D_f = [-2, 2] \cup (4, +\infty)$

P.3. $|2x-5| = \begin{cases} 2x-5 & \text{iff } 2x-5 \geq 0 \text{ iff } x \geq 5/2 \\ -2x+5 & \text{iff } 2x-5 < 0 \text{ iff } x < 5/2 \end{cases}$

So, $f(x) = 3 + |2x-5| = \begin{cases} 3+2x-5 & \text{if } x \geq 5/2 \\ 3-2x+5 & \text{if } x < 5/2 \end{cases}$

$f(x) = \begin{cases} 2x-2 & \text{if } x \geq 5/2 \\ 8-2x & \text{if } x < 5/2 \end{cases}$ is piecewise defined with a breakpoint at $x = 5/2$.

P4. If $f(x) = \frac{1+x}{1-x}$, $g(x) = \frac{x}{1-x}$.

then $f \circ g(x) = f(g(x)) = \frac{1+g(x)}{1-g(x)}$

$$= \frac{1 + \frac{x}{1-x}}{1 - \frac{x}{1-x}} = \frac{\frac{1-x+x}{1-x}}{\frac{1-x-x}{1-x}}$$

$$\boxed{f \circ g(x) = \frac{1}{1-2x}}$$

$D_f = \mathbb{R} \setminus \{1\}$, $D_g = \mathbb{R} \setminus \{1\}$

$D_{f\circ g} = \{x \in D_g : g(x) \in D_f\}$

$= \{x \in \mathbb{R}\setminus\{1\} : \frac{x}{1-x} \in \mathbb{R}\setminus\{1\}\}$

$= \{x \in \mathbb{R}\setminus\{1\} : \frac{x}{1-x} \neq 1\} = \{x \in \mathbb{R}\setminus\{1\} : x \neq 1-x\}$

$= \{x \in \mathbb{R}\setminus\{1\} : x \neq 1/2\} = \mathbb{R}\setminus\{1, 1/2\}$

$= (-\infty, 1/2) \cup (1/2, 1) \cup (1, +\infty)$.

Or: $D_{f\circ g} = D_g \cap D\left(\frac{1}{1-2x}\right) = (\mathbb{R}\setminus\{1\}) \cap (\mathbb{R}\setminus\{1/2\})$

$= \mathbb{R}\setminus\{1, 1/2\} = (-\infty, 1/2) \cup (1/2, 1) \cup (1, +\infty)$