

Here are some short answers for some of the problems on the review list. If you do not know how to solve some of the problems, make sure you talk to someone in the continuous tutorial or your TA or come and see me.

1. (a) False.
(b) True.
(c) False.
(d) $122/9$.
(e) $\ln(e + e^{-1}) - \ln 2$.
(f) $-2(\operatorname{arcsinh} 2 + \sqrt{2}) - \sqrt{2}$. The integral is nasty and it may be left undone since we did not discuss these sort of integrals in class. The point of the problem is that you can pull the x out of the integral and then use the product rule and the fundamental theorem of calculus to do the calculation.
(g) The interval $(-1 - \sqrt{2}, -1 + \sqrt{2})$.
(h) $1.5/3600$ kilometers per second per second.
(i) By L'Hôpital, the limit is 0.
2. $\sqrt{45}$.
3. 1.
4. This is a straight verification.
5. $\sqrt{13} \approx 4 + \frac{1}{8}3 - \frac{1}{256}3^2 + \frac{3}{2048}3^3$.
6. The two numbers are $\pm\sqrt{12}$.
7. $(ay - b)/(bx - a) = -1$.
8. This is straightforward.
9. $(a, b) = (6, 2)$.
10. Construct a continuous function for the time difference and use the intermediate value theorem.
11. $-11\pi/12$ radians per hour.
12. If A is the area, $(r, \phi) = (\sqrt{A}, 2)$.

13. $y = \pm x/\sqrt{3}$.

14. If D denotes the distance, then

$$2D \frac{dD}{dt} = 100 + 96 \sin(b - a) \left(\frac{db}{dt} - \frac{da}{dt} \right)$$

with $a = 2\pi/36$, $b = 2\pi/3$, $\frac{db}{dt} = 2\pi$, $\frac{da}{dt} = 2\pi/12$, with a the angle of the hour hand, b the angle of the minute hand.

15. Let S be the surface area. Then the radius and height are $(r, h) = (\sqrt{S/(6\pi)}, \sqrt{2S/(3\pi)})$.

16. 289 feet.

17. $x = -(ad - bc)/(dy - b)^2$. The derivative is now easy to find.

18. Write

$$\int_1^{ab} \frac{1}{t} dt = \int_1^a \frac{1}{t} dt + \int_a^{ab} \frac{1}{t} dt$$

and make the change of variable $u = t/a$ in the last integral.

19. This follows immediately from the definition.

20. $y = x^3$ is the most famous example.

21. $4/3$.

22. $-5/2$.

23. The derivative is 1.

24. $y = \pm 1$.

25. $x = (-1 \pm \sqrt{29})/2$. The tangents are parallel there.

26. The width W and the depth D are $(W, D) = (12/\sqrt{3}, 12\sqrt{23})$.

27. The function $x - \cos x$ has a non-negative derivative and is negative at $x = 0$ and positive at $x = 2$.

28. This follows by induction with the product rule.

29. This is even easier if you write $y = x^2 + 1/x$.

30. This is for free because you have these already memorized, right?

31. This is straightforward algebra.
32. The algebra becomes simple if you observe that $x = 0$ is a critical point.
33. The line $x = 2$ is a vertical asymptote, and $y = x/2 + 1$ is a slant asymptote.
34. The ratio of the height to the radius is 2:1, and the radius of the cylinder is $r = (2\pi)^{-1/3}$.
35. The volume is $16\pi/3$.
36. Let h be the height of the window, $2w$ the width, and P the perimeter. Then $w = 2P/(8 + 3\pi)$.
37. 67 people.
38. The deceleration is $3 \cdot 88^2/(2 \cdot 242)$.
39. The area is two thirds of the base times the height, $A = 2bh/3$.
40. (a) True.
 (b) True.
 (c) True.
 (d) True.
 (e) False.
 (f) False.
 (g) True.
41. (a) $f(4) = 1/2$.
 (b) $f(4) = \sqrt[3]{12}$.
42. In fact, if you stare at the proof for a moment you will realize that tangent to p at the midpoint of any two of the roots intersects the graph of p at the third root. This is a very beautiful property of cubic curves. The easiest approach is just to write $p(x) = (x - a_1)(x - a_2)(x - a_3) = x^3 - \sigma_1 x^2 + \sigma_2 x - \sigma_3$, where the σ s are the elementary symmetric functions of the roots $\sigma_1 = a_1 + a_2 + a_3$, $\sigma_2 = a_1 a_2 + a_2 a_3 + a_3 a_1$, and $\sigma_3 = a_1 a_2 a_3$.