

1. Answer the following true or false (2 points each).
  - (a) The set of all values of the slope of the line  $y = mx + 2$  for which the  $x$ -intercept exceeds  $1/2$  is  $m > 4$ .
  - (b) The following limit is  $-1.5$

$$\lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^3 - 5x^2 + 7x - 3}$$

- (c) The equation  $\cos x = x$  has at least one solution.
    - (d) If  $f$  and  $g$  are both continuous at  $0$ , then the composite function  $f \circ g(x) = f(g(x))$  is continuous at  $0$ .
2. Compute the following derivatives (2 points each)

- (a)  $y'(2)$  if  $y = \sqrt[3]{x + \sqrt{x}}$ .
  - (b)  $y'(\pi/4)$  if  $y = \sin(\tan x)$ .
  - (c)  $y'(1)$  if  $y = e^{3x} \ln \sqrt{x}$ .

3. The cissoid of Diocles is the curve given by  $y^2(2 - x) = x^3$ . The normal line to a curve at the point  $p$  is the line passing through  $p$  that is perpendicular to the tangent line to the curve at  $p$ . Find both the tangent line and the normal line to the cissoid at  $(x, y) = (1, 1)$ . (5 points)
4. The velocity of a heavy meteorite entering the earth's atmosphere is inversely proportional to  $\sqrt{s}$  when it is  $s$  kilometers from the earth's center. Show that the meteorite's acceleration is inversely proportional to  $s^2$ . (5 points)
5. Starting at 4 am, a monk slowly climbed to the top of a mountain, arriving at noon. The next day, he returned along the same path, starting at 5 am and getting to the bottom at 11 am. Show that at some point along the path, his watch showed the same time on both days.