THE UNIVERSITY OF CALGARY
DEPARTMENT OF MATHEMATICS AND STATISTICS
SAMPLE FINAL EXAM
MATH 251

I.D. NUMBER	SURNAME	OTHER NAMES

STUDENT IDENTIFICATION

Each candidate must sign the Seating List confirming presence at the examination. All candidates for final examinations are required to place their University of Calgary student I.D. cards on their desks for the duration of the examination. (Students writing mid-term tests can also be asked to provide identity proof.) Students without an I.D. card who can produce an **acceptable** alternative I.D., e.g., one with a printed name and photograph, are allowed to write the examination.

A student without acceptable I.D. will be required to complete an Identification Form. The form indicates that there is no guarantee that the examination paper will be graded if any discrepancies in identification are discovered after verification with the student's file. **A student who refuses to produce identification or who refuses to complete and sign the Identification Form is not permitted to write the examination.**

EXAMINATION RULES

1. Students late in arriving will not normally be admitted after one-half hour of the examination time has passed.
2. No candidate will be permitted to leave the examination room until one-half hour has elapsed after the opening of the examination, nor during the last 15 minutes of the examination. All candidates remaining during the last 15 minutes of the examination period must remain at their desks until their papers have been collected by an invigilator.
3. All inquiries and requests must be addressed to supervisors only.
4. **Candidates are strictly cautioned against**:
   (a) speaking to other candidates or communicating with them under any circumstances whatsoever;
   (b) bringing into the examination room any textbook, notebook or memoranda not authorized by the examiner;
   (c) making use of calculators and/or portable computing machines not authorized by the instructor;
   (d) leaving answer papers exposed to view;
   (e) attempting to read other students' examination papers.

   The penalty for violation of these rules is suspension or expulsion or such other penalty as may be determined.
5. Candidates are requested to write on both sides of the page, unless the examiner has asked that the left half page be reserved for rough drafts or calculations.
6. Discarded matter is to be struck out and not removed by mutilation of the examination answer book.
7. Candidates are cautioned against writing in their answer book any matter extraneous to the actual answering of the question set.
8. The candidate is to write his/her name on each answer book as directed and is to number each book.
9. During the examination a candidate must report to a supervisor before leaving the examination room.
10. Candidates must stop writing when the signal is given. Answer books must be handed to the supervisor-in-charge promptly. Failure to comply with these regulations will be cause for rejection of an answer paper.
11. If during the course of an examination a student becomes ill or receives word of domestic affliction, the student must report at once to the supervisor, hand in the unfinished paper and request that it be canceled. If physical and/or emotional ill health is the cause, the student must report at once to a physician/counselor so that subsequent application for a deferred examination is supported by a completed Physical/Counselor Statement form. Students can consult professionals at University Health Services or Counseling and Student Development Centre during normal working hours or consult their physician/counselor in the community. **Once an examination has been handed in for marking a student cannot request that the examination be canceled for whatever reason. Such a request will be denied. Retroactive withdrawals will also not be considered.**

Question	Total Marks	Actual Marks
Total		

NOTE: A calculator and formula sheet *are not* allowed.

Please show all your work.

1) Find the horizontal and vertical asymptotes (if they exist) of the function

$$f(x) = \frac{\sqrt{x^2+8}-3}{x-1}.$$

2) Differentiate, but do not simplify the following functions

a)

$$f(x) = \frac{(x^2-8)^{\frac{1}{3}}}{\tan(e^x)},$$

b)

$$h(x) = \sin\left(x^2+1\right)^x.$$

3) Find the constant k so that the function:

$$f(x) = \begin{cases} \frac{x^2-9}{x+3}, & x \neq -3 \\ kx+6, & x = -3 \end{cases},$$

is continuous everywhere.

4) Find an equation of the tangent line to the curve $x^2+xy-y^2=1$, at the point $(2,3)$.

5) A water tank is in the shape of an inverted right circular cone with dept 5 m and the top radius 2 m. Water is draining out of the bottom of the tank at a rate of $\frac{1}{2}$ $\mathrm{m}^3/\mathrm{min}$. Find the rate at which the water level is decreasing when the water in the tank is 4 m deep. (**Note:** The volume of a right circular cone is $V = \frac{\pi r^2 h}{3}$.)

6) Using an appropriate linear approximation estimate $\sqrt[4]{82}$.

7) Find the absolute minimum and maximum of the function

$$f(x) = \left(4-x^2\right)^2, \quad \text{on the interval } [-1,3].$$

8) A cylindrical can, open at top, is to hold 500 cm^3 of liquid. Find the height and the radius that minimize the amount of material needed to manufacture the can.

9) For the function $f(x) = \frac{x^2-1}{x^3}$ find:

a) Horizontal, vertical and oblique asymptotes if they exist;

b) Intervals of increase, decrease and relative extrema;

c) Intervals where the function is concave up, down, and the inflection points if they exist;

d) Sketch the graph.

10) Evaluate the integrals

a)

$$\int \frac{\cos(\sqrt{x})\sin(\sqrt{x})}{\sqrt{x}}dx \ ,$$

b)

$$\int_1^e \frac{(1+\ln x)^2}{x}dx,$$

c)

$$\int_0^{\frac{\pi}{4}} \sqrt{\tan x}\sec^2 x dx.$$

11) Sketch the region enclosed by the curves $y = x^2$ and $y = -x^2 + 2x$, and find its area.

End of Examination