University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 251

Worksheet 7
[Introduction to derivatives]

1. In each case, given f(x), determine $\lim_{h \to 0} \left(\frac{f(x + h) - f(x)}{h} \right)$

   a. $f(x) = x^3$

   b. $f(x) = \frac{1}{x^2}$

   c. $f(x) = \sqrt{x}$

   d. $f(x) = \frac{x + 1}{x + 2}$

2. Find i. the slope of the tangent line to the given curve at the given point,
   ii. the equation of the tangent line to the curve at the given point,
   iii. the equation of the normal line to the curve at the given point.

   a. $y = x^2 - 3x$ at $(1 , -2)$

   b. $y = \frac{2 + 3x}{4 - 3x}$ at $\left(0 , \frac{1}{2}\right)$

   c. $y = \sqrt{4 - x^2}$ at $\left(1, \sqrt{3}\right)$

   d. $y = \sin(2x)$ at $\left(\frac{\pi}{6}, \frac{\sqrt{3}}{2}\right)$

   e. $y = \sec x$ at $\left(\frac{\pi}{4}, \sqrt{2}\right)$

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3. Use the definition of the derivative as a limit to determine $f'(x)$ given that

a. $f(x) = \sqrt{x^2 + x}$

b. $f(x) = \dfrac{1}{x^2 - 4}$

c. $f(x) = \dfrac{4x + 5}{3 - 5x}$

d. $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$

e. $f(x) = \cos x$

f. $f(x) = csc\ x$

4. For $f(x)$ and a given below, determine $f'(a)$ if it exists. If it does not exist explain why.

a. $f(x) = |x|$ when $a = 0$

b. $f(x) = x^{\frac{2}{3}}$ when $a = 0$

c. $f(x) = |4 - x^2|$ when $a = 2$

d. $f(x) = |4 - x^2|$ when $a = -2$

5. In each case, determine $f'(x)$ given $f(x)$.

i. $f(x) = \sin\left(x^4 + x^3\right)$

ii. $f(x) = \tan\left(\sqrt{4 - x^3}\right)$

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iii. $f(x) = \sec\left(x^{2/3} - x\right)$

iv. $f(x) = csc\left(\sqrt{x^4 + 1}\right)$

v. $f(x) = \cos\left(\cos\left(\cos x\right)\right)$

vi. $f(x) = \cot\left(\sin\left(x^3 + 1\right)\right)$

vii. $f(x) = \sqrt{x^2 + \sqrt{x^2 + \sqrt{x^2 + 1}}}$

6. Determine all points on each graph of the function given where the tangent line is parallel to the x-axis.

a. $y = x^4 - x^2 + 1$

b. $y = \sin 2x - 2 \sin x$

c. $y = \tan x + \cot x$

d. $y = 2x^3 - 9x^2 - 24x + 4$

8. Determine whether or not the given curve has a tangent line which is parallel to the x-axis:

a. $y = 2x^3 + 3x^2 + 6x + 12$

b. $y = 2x^3 - x^2 + 2x - 1$