University of Calgary
Faculty of Science
Department of Mathematics and Statistics

Math 251

Worksheet 8
[Implicit Differentiation with applications]

1. Determine a point on the curve $y = \sqrt{1 - x^2}$ at which the tangent line will have slope equal to 1.

2. In each case determine the equation of the tangent and normal lines of the curve given at the given point.
   a. $x^3 + y^3 = 9$ at the point (2,1).

   b. $x^4 - x^3y + 2xy^2 - 5 = 0$ at the point (-1,2).

   c. $4x^2 - 3xy + 3y^2 = 25$ at the point (-2,1).

   d. $x^2 + xy + 2y^3 = 4$ at the point (-2,1).

   e. $\tan(xy^2) = \frac{2xy}{\pi}$ at the point $\left(-\pi, \frac{1}{2}\right)$.

3. Given the curve $y(y^2 - 1)(y - 2) = x(x - 1)(x - 2)$.
   a. Determine the x-coordinates of the points on the curve where the tangent line is parallel to the x-axis.
   b. Find the equation of the tangent line to the curve at (0,1) and at (0,2).

4. Two curves are orthogonal exactly when the tangents to each curve at the point of intersection of the curves are at right angles.
   a. Show that the curves $x^2 - y^2 = 5$ and $4x^2 + 9y^2 = 72$ are orthogonal.

   b. Show that $x^2 + y^2 = ax$ and $x^2 + y^2 = by$ are orthogonal families of curves.

5. Consider the curve $\sqrt{x} + \sqrt{y} = \sqrt{c}$. Show that the sum of the x-and y-intercepts of any tangent line to the curve is equal to c.